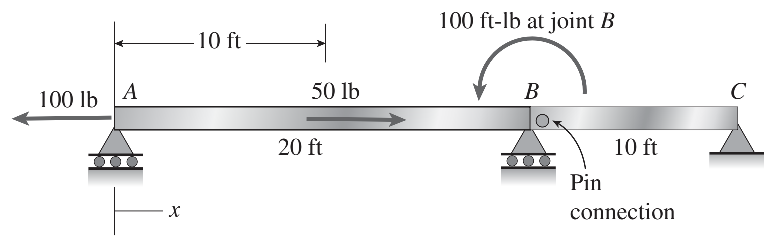
1

Tension, Compression, and Shear

Statics Review

Problem 1.2-1 Segments AB and BC of beam ABC are pin connected a small distance to the right of joint B (see figure). Axial loads act at A and at mid-span of AB. A concentrated moment is applied at joint B.

(a) Find reactions at supports A, B, and C.
(b) Find internal stress resultants N, V, and M at $x = 15$ ft.



Solution 1.2-1

(a) Apply laws of statics

$\Sigma F_x = 0 \qquad C_x = 100\text{ lb} - 50\text{ lb} = 50\text{ lb}$

FBD of BC $\qquad \Sigma M_B = 0 \qquad C_y = \dfrac{1}{10\text{ ft}}(0) = 0$

Entire FBD $\qquad \Sigma M_A = 0 \qquad B_y = \dfrac{1}{20\text{ ft}}(-100\text{ lb-ft}) = -5\text{ lb}$

$\Sigma F_y = 0 \qquad A_y = -B_y = 5\text{ lb-ft}$

Reactions are $\boxed{A_y = 5\text{ lb}}$ $\boxed{B_y = -5\text{ lb}}$ $\boxed{C_x = 50\text{ lb}}$ $\boxed{C_y = 0}$

(b) Internal stress resultants N, V, and M at $x = 15$ ft

Use FBD of segment from A to $x = 15$ ft

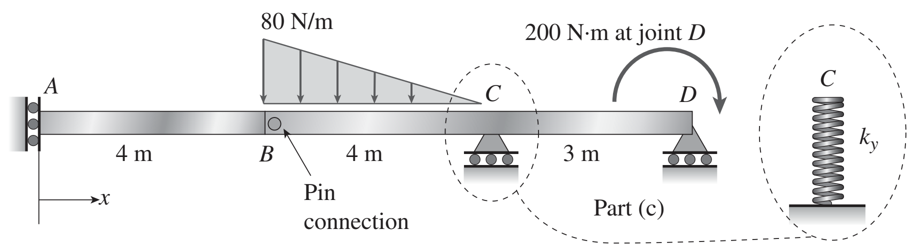
$\Sigma F_x = 0 \qquad \boxed{N = 100\text{ lb} - 50\text{ lb} = 50\text{ lb}}$

$\Sigma F_y = 0 \qquad \boxed{V = A_y = 5\text{ lb}}$

$\Sigma M = 0 \qquad \boxed{M = A_y 15\text{ ft} = 75\text{ lb-ft}}$

Problem 1.2-2 Segments AB and BCD of beam $ABCD$ are pin connected at $x = 4$ m. The beam is supported by a sliding support at A and roller supports at C and D (see figure). A triangularly distributed load with peak intensity of 80 N/m acts on BC. A concentrated moment is applied at joint B.

(a) Find reactions at supports A, C, and D.
(b) Find internal stress resultants N, V, and M at $x = 5$ m.
(c) Repeat parts (a) and (b) for the case of the roller support at C replaced by a linear spring of stiffness $k_y = 200$ kN/m.



Solution 1.2-2

(a) Apply laws of statics

$\Sigma F_x = 0 \qquad A_x = 0$

FBD of $AB \qquad \Sigma M_B = 0 \qquad M_A = 0$

Entire FBD $\qquad \Sigma M_C = 0 \qquad D_y = \frac{1}{3\text{ m}}\left[200\text{ N}\cdot\text{m} - \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{2}{3}\right)4\text{ m}\right] = -75.556\text{ N}$

$\Sigma F_y = 0 \qquad C_y = \frac{1}{2}(80\text{ N/m})\,4\text{ m} - D_y = 235.556\text{ N}$

Reactions are $\boxed{M_A = 0} \qquad A_x = 0 \qquad \boxed{C_y = 236\text{ N}} \qquad \boxed{D_y = -75.6\text{ N}}$

(b) Internal stress resultants N, V, and M at $x = 5$ m

Use FBD of segment from A to $x = 5$ m; ordinate on triangular load at $x = 5$ m is $\frac{3}{4}(80\text{ N/m}) = 60\text{ N/m}$.

$\Sigma F_x = 0 \qquad N_x = -A_x = 0$

$\Sigma F_y = 0 \qquad V = \frac{-1}{2}[(80\text{ N/m} + 60\text{ N/m})\,1\text{ m}] = -70\text{ N} \qquad \boxed{V = -70\text{ N}}$ Upward

$\Sigma M = 0 \qquad M = -M_A - \frac{1}{2}(80\text{ N/m})\,1\text{ m}\left(\frac{2}{3}1\text{ m}\right) - \frac{1}{2}(60\text{ N/m})\,1\text{ m}\left(\frac{1}{3}1\text{ m}\right) = -36.667\text{ N}\cdot\text{m}$

(break trapezoidal load into two triangular loads in moment expression)

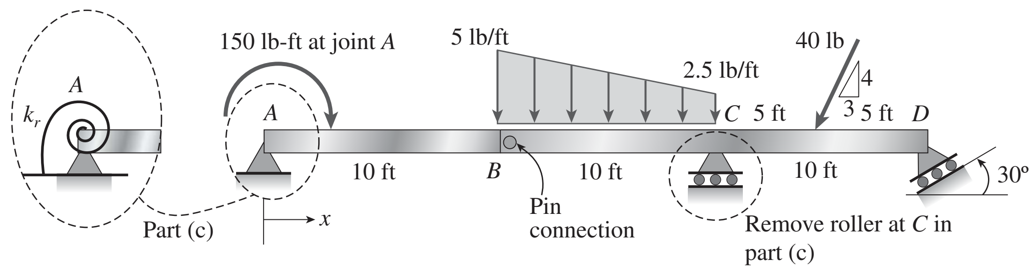
$\boxed{M = -36.7\text{ N}\cdot\text{m}}$ CW

(c) Replace roller support at C with spring support

Structure remains statically determinate so all results above in (a) and (b) are unchanged.

Problem 1.2-3 Segments *AB* and *BCD* of beam *ABCD* are pin connected at $x = 10$ ft. The beam is supported by a pin support at *A* and roller supports at *C* and *D*; the roller at *D* is rotated by 30° from the *x* axis (see figure). A trapezoidal distributed load on *BC* varies in intensity from 5 lb/ft at *B* to 2.5 lb/ft at *C*. A concentrated moment is applied at joint *A* and a 40 lb inclined load is applied at mid-span of *CD*.

(a) Find reactions at supports *A*, *C*, and *D*.
(b) Find the resultant force in the pin connection at *B*.
(c) Repeat parts (a) and (b) if a rotational spring ($k_r = 50$ ft-lb/rad) is added at *A* *and* the roller at *C* is removed.



Solution 1.2-3

(a) STATICS

FBD of *AB* (cut through beam at pin): $\Sigma M_B = 0 \qquad A_y = \dfrac{1}{10\text{ ft}}(-150\text{ lb-ft}) = -15\text{ lb}$

Entire FBD: $\Sigma M_D = 0$

$$C_y = \frac{1}{10\text{ ft}}\left[\frac{4}{5}40\text{ lb}(5\text{ ft}) + \frac{1}{2}(2.5\text{ lb/ft})10\text{ ft}\left(10\text{ ft} + \frac{10\text{ ft}}{3}\right) + \frac{1}{2}(5\text{ lb/ft})10\text{ ft}\left(10\text{ ft} + \frac{2}{3}10\text{ ft}\right)\right.$$

$$\left. -\ 150\text{ lb-ft} - A_y\,30\text{ ft}\right] = 104.333\text{ lb}$$

$$\Sigma F_y = 0 \qquad D_y = \frac{4}{5}40\text{ lb} + \frac{1}{2}(5\text{ lb/ft} + 2.5\text{ lb/ft})\,10\text{ ft} - A_y - C_y = -19.833\text{ lb} \quad \text{so} \quad D_x = \frac{-D_y}{\tan(60°)} = 11.451\text{ lb}$$

$$\Sigma F_x = 0 \qquad A_x = \frac{3}{5}40\text{ lb} - D_x = 12.549\text{ lb}$$

$$\boxed{A_x = 12.55\text{ lb},\ A_y = -15\text{ lb},\ C_y = 104.3\text{ lb},\ D_x = 11.45\text{ lb},\ D_y = -19.83\text{ lb}}$$

(b) USE FBD OF *AB* ONLY; MOMENT AT PIN IS ZERO

$$F_{Bx} = -A_x \qquad F_{Bx} = -12.55\text{ lb} \qquad F_{By} = -A_y \qquad F_{By} = 15\text{ lb} \qquad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^{\,2} + F_{By}^{\,2}} = 19.56\text{ lb}}$$

(c) ADD ROTATIONAL SPRING AT *A* AND REMOVE ROLLER AT *C*; APPLY EQUATIONS OF STATICAL EQUILIBRIUM

Use FBD of *BCD* $\Sigma M_B = 0$

$$D_y = \frac{1}{20\text{ ft}}\left[\frac{1}{2}(2.5\text{ lb/ft})\,10\text{ ft}\left(\frac{2}{3}10\text{ ft}\right) + \frac{1}{2}(5\text{ lb/ft})\,10\text{ ft}\left(\frac{1}{3}10\text{ ft}\right) + \frac{4}{5}40\text{ lb}\,(15\text{ ft})\right] = 32.333\text{ lb}$$

$$\text{so} \qquad D_x = \frac{-D_y}{\tan(60°)} = -18.668\text{ lb}$$

Use entire FBD $\qquad \Sigma F_y = 0 \qquad A_y = \dfrac{1}{2}(5\text{ lb/ft} + 2.5\text{ lb/ft})\,10\text{ ft} + \dfrac{4}{5}(40\text{ lb}) - D_y = 37.167\text{ lb}$

$$\Sigma F_x = 0 \qquad A_x = \frac{3}{5}(40\text{ lb}) - D_x = 42.668\text{ lb}$$

Use FBD of AB $\quad \Sigma M_B = 0 \quad M_A = 150\text{ lb-ft} + A_y 10\text{ ft} = 521.667\text{ lb-ft}$

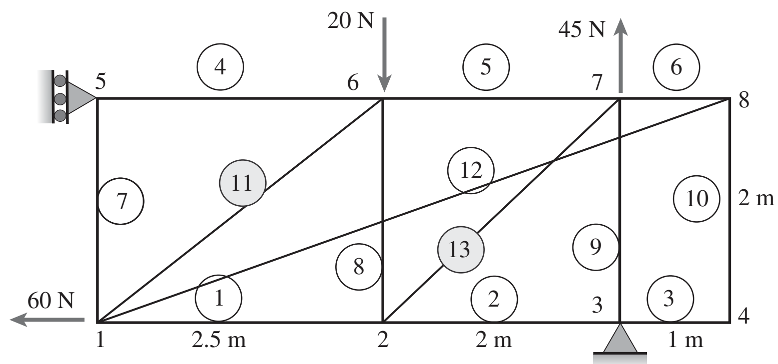
SO REACTIONS ARE $\boxed{A_x = 42.7\text{ lb}}$ $\boxed{A_y = 37.2\text{ lb}}$ $\boxed{M_A = 522\text{ lb-ft}}$ $\boxed{D_x = -18.67\text{ lb}}$ $\boxed{D_y = 32.3\text{ lb}}$

RESULTANT FORCE IN PIN CONNECTION AT B

$$F_{Bx} = -A_x \qquad F_{By} = -A_y \qquad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^{\;2} + F_{By}^{\;2}} = 56.6\text{ lb}}$$

Problem 1.2-4 Consider the plane truss with a pin support at joint 3 and a vertical roller support at joint 5 (see figure).

(a) Find reactions at support joints 3 and 5.
(b) Find axial forces in truss members 11 and 13.



Solution 1.2-4

(a) STATICS

$$\Sigma F_y = 0 \qquad R_{3y} = 20\text{ N} - 45\text{ N} = -25\text{ N}$$

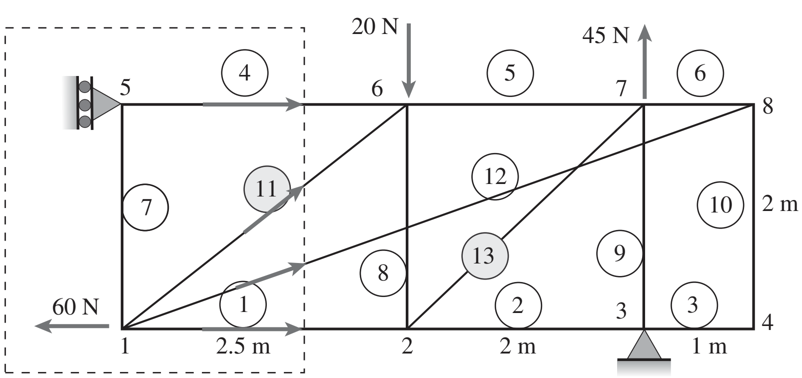
$$\Sigma M_3 = 0 \qquad R_{5x} = \frac{1}{2\text{ m}}(20\text{ N} \times 2\text{ m}) = 20\text{ N}$$

$$\Sigma F_x = 0 \qquad R_{3x} = -R_{5x} + 60\text{ N} = 40\text{ N}$$

(b) MEMBER FORCES IN MEMBERS 11 and 13

Number of unknowns: $m = 13 \quad r = 3 \quad m + r = 16$

Number of equations: $j = 8 \quad 2j = 16$ So statically determinate



Section cut for left FBD

TRUSS ANALYSIS

(1) $\Sigma F_V = 0$ at joint 4 so $F_{10} = 0$

(2) $\Sigma F_V = 0$ at joint 8 so $F_{12} = 0$

(3) $\Sigma F_H = 0$ at joint 5 so $F_4 = -R_{5x} = -20\text{ N}$

(4) Cut vertically through 4, 11, 12, and 1; use left FBD; sum moments about joint 2

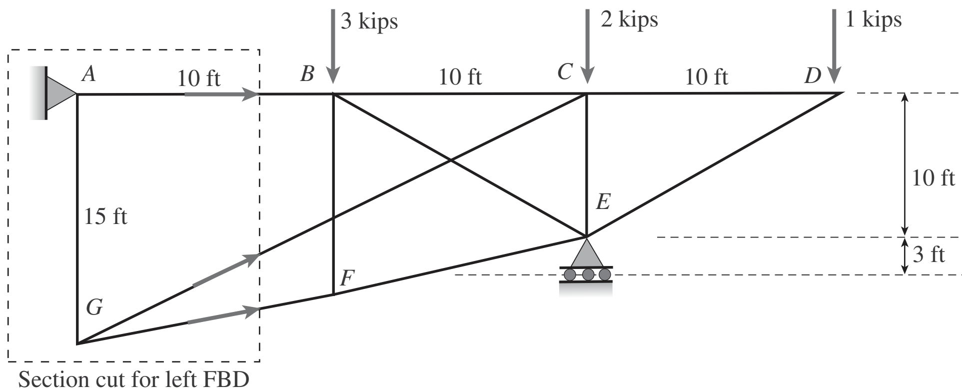
$$F_{11V} = \frac{1}{2.5\text{ m}}(R_{5x} - F_4) \quad \text{so} \quad \boxed{F_{11} = 0}$$

(5) Sum vertical forces at joint 3; $F_9 = R_{3y}$
$F_9 = 25\text{ N}$

(6) Sum vertical forces at joint 7 $\quad F_{13V} = 45\text{ N} - F_9 = 20\text{ N} \quad \boxed{F_{13} = \sqrt{2}F_{13V} = 28.3\text{ N}}$

Problem 1.2-5 A plane truss has a pin support at A and a roller support at E (see figure).

(a) Find reactions at all supports.
(b) Find the axial force in truss member FE.



Solution 1.2-5

(a) STATICS

$$\Sigma F_x = 0 \qquad A_x = 0$$

$$\Sigma M_A = 0 \qquad E_y = \frac{1}{20\text{ ft}}(3\text{ k} \times 10\text{ ft} + 2\text{ k} \times 20\text{ ft} + 1\text{ k} \times 30\text{ ft}) = 5\text{ k}$$

$$\Sigma F_y = 0 \qquad A_y = 3\text{ k} + 2\text{ k} + 1\text{ k} - E_y = 1\text{ k}$$

(b) MEMBER FORCE IN MEMBER FE

Number of unknowns: $m = 11$ $r = 3$ $m + r = 14$

Number of equations: $j = 7$ $2j = 14$ So statically determinate

TRUSS ANALYSIS

(1) Cut vertically through AB, GC, and GF; use left FBD; sum moments about C

$$F_{GFx}(15\text{ ft}) - F_{GFy}(20\text{ ft}) = A_y(20\text{ ft}) = 20\text{ ft-k} \qquad F_{GFx} = F_{GF}\frac{10}{\sqrt{2^2 + 10^2}} \qquad F_{GFy} = F_{GF}\frac{2}{\sqrt{2^2 + 10^2}}$$

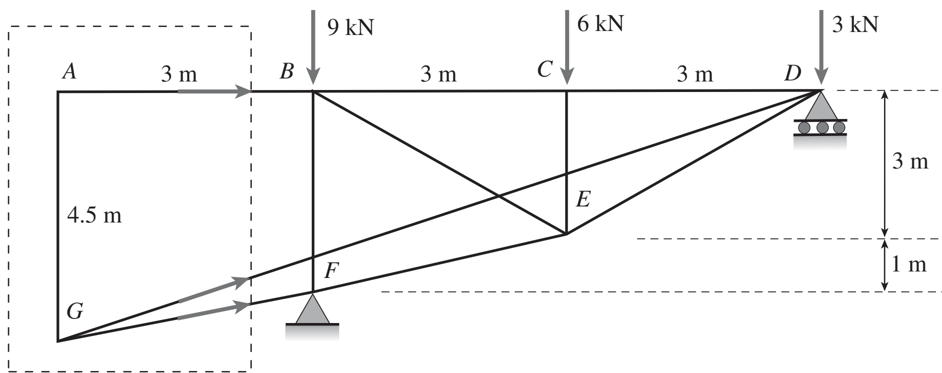
so $$F_{GF} = \frac{A_y(20\text{ ft})}{15\text{ ft}\dfrac{10}{\sqrt{2^2 + 10^2}} - 20\text{ ft}\dfrac{2}{\sqrt{2^2 + 10^2}}} = 1.854\text{ k} \quad \text{and} \quad F_{GFx} = F_{GF}\frac{10}{\sqrt{2^2 + 10^2}} = 1.818\text{ k}$$

(2) Sum horizontal forces at joint F $$F_{FEx} = F_{GFx} = 1.818\text{ k} \qquad F_{FE} = \frac{\sqrt{10^2 + 3^2}}{10}F_{FEx} = 1.898\text{ k}$$

$$\boxed{F_{FE} = 1.898\text{ k}}$$

Problem 1.2-6 A plane truss has a pin support at F and a roller support at D (see figure).

(a) Find reactions at both supports.
(b) Find the axial force in truss member FE.



Solution 1.2-6

(a) STATICS

$$\Sigma F_x = 0 \qquad F_x = 0$$

$$\Sigma M_F = 0 \qquad D_y = \frac{1}{6\text{ m}}\left[3\text{ kN}(6\text{ m}) + 6\text{ kN}(3\text{ m})\right] = 6\text{ kN}$$

$$\Sigma F_y = 0 \qquad F_y = 9\text{ kN} + 6\text{ kN} + 3\text{ kN} - D_y = 12\text{ kN}$$

(b) MEMBER FORCE IN MEMBER FE

Number of unknowns: $m = 11$ $r = 3$ $m + r = 14$

Number of equations: $j = 7$ $2j = 14$ So statically determinate

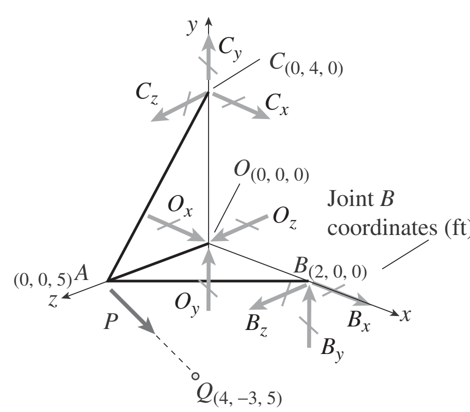
TRUSS ANALYSIS

(1) Cut vertically through AB, GD, and GF; use left FBD; sum moments about D to get $F_{GF} = 0$

(2) Sum horizontal forces at joint F $F_{FEx} = -F_x = 0$ so $\boxed{F_{FE} = 0}$

Problem 1.2-7 A space truss has three-dimensional pin supports at joints O, B, and C. Load P is applied at joint A and acts toward point Q. Coordinates of all joints are given in feet (see figure).

(a) Find reaction force components B_x, B_z, and O_z.
(b) Find the axial force in truss member AC.



Solution 1.2-7

(a) FIND REACTIONS USING STATICS $\quad m = 3 \quad r = 9 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ So truss is statically determinate

$$r_{AQ} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \quad r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.8 \\ -0.6 \\ 0 \end{pmatrix} \quad P_A = P e_{AQ} = \begin{pmatrix} 0.8P \\ -0.6P \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma M = 0$

$$M_O = r_{OA} \times P_A + r_{OC} \times \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + r_{OB} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 4C_z + 3.0P \\ 4.0P - 2B_z \\ 2B_y - 4C_x \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_z = \frac{-3}{4}P$$

$$\Sigma M_y = 0 \quad \text{gives} \quad \boxed{B_z = 2P}$$

$\Sigma F = 0$

$$R_O = P_A + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} B_x + C_x + O_x + 0.8P \\ B_y + C_y + O_y + -0.6P \\ O_z + \frac{5P}{4} \end{pmatrix} \quad \text{so} \quad \Sigma M_z = 0 \quad \text{gives} \quad \boxed{O_z = \frac{-5}{4}P}$$

METHOD OF JOINTS $\quad$ Joint O $\quad \Sigma F_x = 0 \quad O_x = 0 \quad \Sigma F_y = 0 \quad O_y = 0$

Joint B $\quad \Sigma F_y = 0 \quad B_y = 0$

Joint C $\quad \Sigma F_x = 0 \quad C_x = 0$

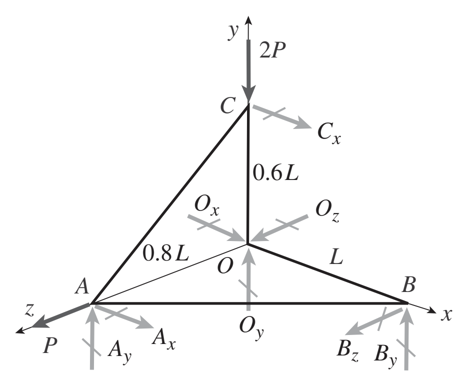
For entire structure $\quad \Sigma F_x = 0$ gives $\boxed{B_x = -0.8P} \quad \Sigma F_y = 0 \quad C_y = 0.6P - B_y = O_y \quad C_y = 0.6P$

(b) FORCE IN MEMBER AC

$$\Sigma F_z = 0 \quad \text{at joint } C \quad F_{AC} = \frac{\sqrt{4^2 + 5^2}}{5}|C_Z| = \frac{3\sqrt{41}\,|P|}{20} \quad \boxed{F_{AC} = \frac{3\sqrt{41}}{20}P} \quad \text{tension} \quad \frac{3\sqrt{41}}{20} = 0.96$$

Problem 1.2-8 A space truss is restrained at joints O, A, B, and C, as shown in the figure. Load P is applied at joint A and load $2P$ acts downward at joint C.

(a) Find reaction force components A_x, B_y, and B_z in terms of load variable P.
(b) Find the axial force in truss member AB in terms of load variable P.



Solution 1.2-8

(a) FIND REACTIONS USING STATICS $m = 4$ $r = 8$ $m + r = 12$ $j = 4$ $3j = 12$

$m + r = 3j$ so truss is statically determinate

$$r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 0.8L \end{pmatrix} \quad r_{OB} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0.6L \\ 0 \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ P \end{pmatrix} \quad F_B = \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} \quad F_c = \begin{pmatrix} C_x \\ -2P \\ 0 \end{pmatrix} \quad F_O = \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} -0.8A_yL \\ 0.8A_xL - B_zL \\ B_yL - 0.6C_xL \end{pmatrix} \quad so \quad \Sigma M_x = 0 \quad \text{gives} \quad A_y = 0$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_O + F_A + F_B + F_C = \begin{pmatrix} A_x + C_x + O_x \\ A_y + B_y + O_y - 2P \\ B_z + O_z + P \end{pmatrix}$$

METHOD OF JOINTS Joint O $\Sigma F_z = 0$ $O_z = 0$

so from $\Sigma F_z = 0$ $\boxed{B_z = -P}$ and $\Sigma M_y = 0$ $\boxed{A_x = \dfrac{B_z}{0.8} = -1.25P}$

Joint B $\Sigma F_y = 0$ $\boxed{B_y = 0}$

Joint C $\Sigma F_x = 0$ $C_x = 0$

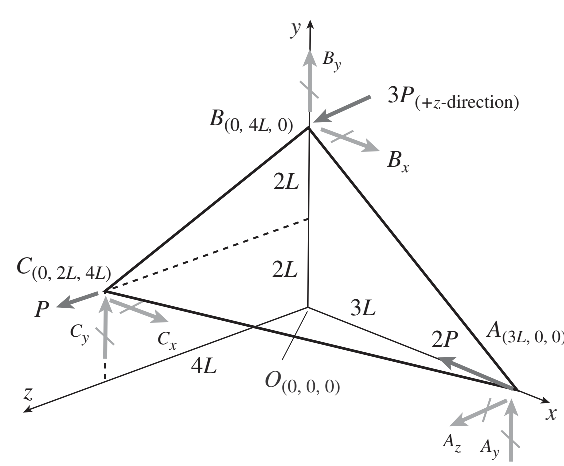
(b) FORCE IN MEMBER AB

$$\Sigma F_z = 0 \quad \text{at joint } B \qquad F_{AB} = \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L}|B_z| \qquad |B_z| = |P| \qquad \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} = 1.601$$

$\boxed{F_{AB} = 1.601P}$ tension

Problem 1.2-9 A space truss is restrained at joints A, B, and C, as shown in the figure. Load $2P$ is applied at in the $-x$ direction at joint A, load $3P$ acts in the $+z$ direction at joint B and load P is applied in the $+z$ direction at joint C. Coordinates of all joints are given in terms of dimension variable L (see figure).

(a) Find reaction force components A_y and A_z in terms of load variable P.
(b) Find the axial force in truss member AB in terms of load variable P.



Solution 1.2-9

(a) Find reactions using statics $\quad m = 3 \quad r = 6 \quad m + r = 9 \quad j = 3 \quad 3j = 9$

$m + r = 3j$ So truss is statically determinate

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 2L \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} -2P \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ B_y \\ 3P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ P \end{pmatrix}$$

$\Sigma M = 0$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} 14LP - 4C_yL \\ 4C_xL - 3A_zL \\ 3A_yL - 4B_xL - 2C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_y = \frac{14}{4}P$$

$\Sigma F = 0$

Resultant force at O

$$R_O = F_A + F_B + F_C = \begin{pmatrix} B_x + C_x - 2P \\ A_y + B_y + C_y \\ A_z + 4P \end{pmatrix} \quad \text{so} \quad \Sigma F_z = 0 \quad \text{gives} \quad \boxed{A_z = -4.0P}$$

Method of joints

Joint A $\quad \Sigma F_z = 0 \quad F_{ACz} = -A_z = 4.0P \quad$ so $\quad F_{ACy} = \frac{2}{4}F_{ACz} = 2.0P \quad F_{ACx} = \frac{3}{4}F_{ACz} = 3.0P$

$\Sigma F_x = 0 \quad F_{ABx} = -2P - F_{ACx} = -3.0P - 2P \quad$ so $\quad F_{ABy} = \frac{4}{3}F_{ABx} = -4.0P - \frac{8P}{3}$

$\Sigma F_y = 0 \quad A_y = -(F_{ABy} + F_{ACy}) = \frac{8P}{3} + 4.0P + -2.0P \quad \boxed{A_y = 4.67P}$

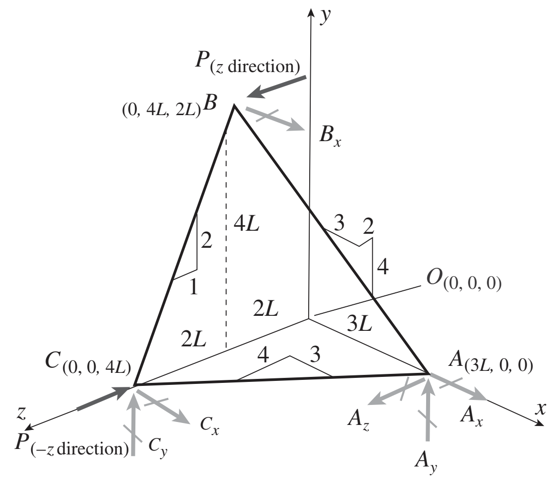
(b) Force in member AB

$$F_{AB} = \sqrt{F_{ABx}^{\ 2} + F_{ABy}^{\ 2}} \qquad F_{AB} = -\sqrt{5^2 + \left(\frac{20}{3}\right)^2}P = -\frac{25P}{3} \qquad \frac{25}{3} = 8.33$$

$\boxed{F_{AB} = -8.33P}$ compression

Problem 1.2-10 A space truss is restrained at joints A, B, and C, as shown in the figure. Load P acts in the $+z$ direction at joint B and in the $-z$ direction at joint C. Coordinates of all joints are given in terms of dimension variable L (see figure). Let $P = 5$ kN and $L = 2$ m.

(a) Find the reaction force components A_z and B_x.
(b) Find the axial force in truss member AB.



Solution 1.2-10

(a) Find reactions using statics $\quad m = 3 \quad r = 6 \quad m + r = 9 \quad j = 3 \quad 3j = 9$

$m + r = 3j$ so truss is statically determinate

$L = 2\text{ m} \qquad P = 5\text{ kN}$

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 2L \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0 \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ 0 \\ P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ -P \end{pmatrix}$$

$\Sigma F = 0$

Resultant force at O $\quad R_O = F_A + F_B + F_C = \begin{pmatrix} A_x + B_x + C_x \\ A_y + C_y \\ A_z \end{pmatrix}$ so $\Sigma F_z = 0$ gives $A_z = 0$

Resultant moment at A

$$r_{AC} = \begin{pmatrix} -3L \\ 0 \\ 4L \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} -0.6 \\ 0 \\ 0.8 \end{pmatrix} \quad r_{AB} = \begin{pmatrix} -3L \\ 4L \\ 2L \end{pmatrix}$$

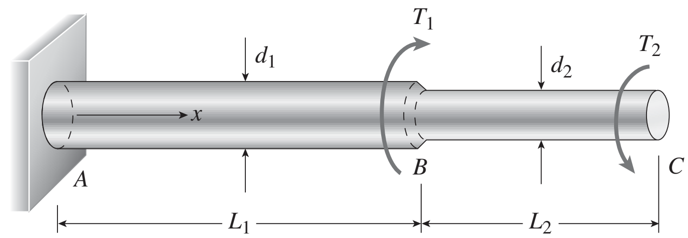
$$M_A = r_{AB} \times F_B + r_{AC} \times F_C = \begin{pmatrix} 120\text{ kN} - 24C_y \\ 12B_x + 24C_x \\ -24B_x - 18C_y \end{pmatrix} \quad M_A e_{AC} = -19.2B_x - 72.0\text{ kN} \quad \text{so} \quad \boxed{B_x = \frac{-72}{19.2}\text{ kN} = -3.75\text{ kN}}$$

(b) Force in member AB

Method of joints at B $\quad \Sigma F_x = 0 \quad F_{ABx} = -B_X \quad \boxed{F_{AB} = \frac{\sqrt{29}}{3}F_{ABx} = 6.73\text{ kN}}$

Problem 1.2-11 A stepped shaft *ABC* consisting of two solid, circular segments is subjected to torques T_1 and T_2 acting in opposite directions, as shown in the figure. The larger segment of the shaft has a diameter of $d_1 = 2.25$ in. and a length of $L_1 = 30$ in.; the smaller segment has a diameter $d_2 = 1.75$ in. and a length $L_2 = 20$ in. The torques are $T_1 = 21{,}000$ lb-in. and $T_2 = 10{,}000$ lb-in.

(a) Find reaction torque T_A at support *A*.
(b) Find the internal torque $T(x)$ at two locations: $x = L_1/2$ and $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams (FBDs).



Solution 1.2-11

(a) APPLY LAWS OF STATICS $L_1 = 30$ in. $L_2 = 20$ in. $T_1 = 21000$ lb-in. $T_2 = 10000$ lb-in.

$\Sigma M_x = 0$ $\boxed{T_A = T_1 - T_2 = 11{,}000 \text{ lb-in.}}$

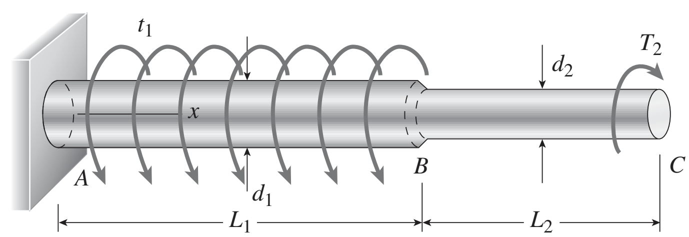
(b) INTERNAL STRESS RESULTANT T AT TWO LOCATIONS

Cut shaft at midpoint between *A* and *B* at $x = L_1/2$ (use left FBD) $\Sigma M_x = 0$ $\boxed{T_{AB} = -T_A = -11{,}000 \text{ lb-in.}}$

Cut shaft at midpoint between *B* and *C* at $x = L_1 + L_2/2$ (use right FBD) $\Sigma M_x = 0$ $\boxed{T_{BC} = T_2 = 10{,}000 \text{ lb-in.}}$

Problem 1.2-12 A stepped shaft *ABC* consisting of two solid, circular segments is subjected to uniformly distributed torque t_1 acting over segment 1 and concentrated torque T_2 applied at *C*, as shown in the figure. Segment 1 of the shaft has a diameter of $d_1 = 57$ mm and length of $L_1 = 0.75$ m; segment 2 has a diameter $d_2 = 44$ mm and length $L_2 = 0.5$ m. Torque intensity $t_1 = 3100$ N·m/m and $T_2 = 1100$ N·m.

(a) Find reaction torque T_A at support *A*.
(b) Find the internal torque *T(x)* at two locations: $x = L_1/2$ and $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams (FBDs).



Solution 1.2-12

(a) REACTION TORQUE AT *A* $L_1 = 0.75$ m $L_2 = 0.75$ m $t_1 = 3100$ N·m/m $T_2 = 1100$ N·m

Statics $\Sigma M_x = 0$ $T_A = -t_1 L_1 + T_2 = -1225$ N·m $\boxed{T_A = -1225 \text{ N·m}}$

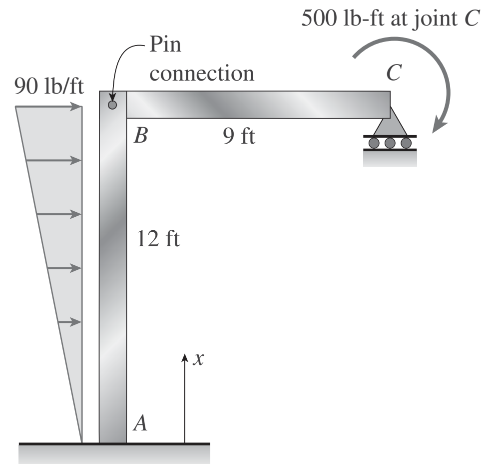
(b) INTERNAL TORSIONAL MOMENTS AT TWO LOCATIONS

Cut shaft between *A* and *B* (use left FBD) $T_1(x) = -T_A - t_1 x$ $\boxed{T_1\left(\frac{L_1}{2}\right) = 62.5 \text{ N·m}}$

Cut shaft between *B* and *C* (use left FBD) $T_2(x) = -T_A - t_1 L_1$ $\boxed{T_2\left(L_1 + \frac{L_2}{2}\right) = -1100 \text{ N·m}}$

Problem 1.2-13 A plane frame is restrained at joints A and C, as shown in the figure. Members AB and BC are pin connected at B. A triangularly distributed lateral load with peak intensity of 90 lb/ft acts on AB. A concentrated moment is applied at joint C.

(a) Find reactions at supports A and C.
(b) Find internal stress resultants N, V, and M at $x = 3$ ft on column AB.



Solution 1.2-13

(a) Statics

$$\Sigma F_H = 0 \qquad A_x = \frac{-1}{2}(90 \text{ lb/ft})\,12 \text{ ft} = -540 \text{ lb}$$

$$\Sigma F_V = 0 \qquad A_y + C_y = 0$$

$$\Sigma M_{\text{FBD}BC} = 0 \qquad C_y = \frac{500 \text{ lb-ft}}{9 \text{ ft}} = 55.6 \text{ lb} \qquad A_y = -C_y = -55.6 \text{ lb}$$

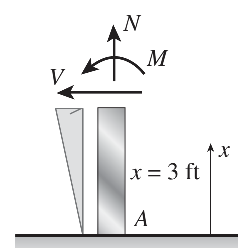
$$\Sigma M_A = 0 \qquad M_A = 500 \text{ lb-ft} + \frac{1}{2}(90 \text{ lb/ft})\,12 \text{ ft}\left(\frac{2}{3}12 \text{ ft}\right) - C_y\,9 \text{ ft} = 4320 \text{ lb-ft}$$

(b) Internal stress resultants

$$N = -A_y = 55.6 \text{ lb}$$

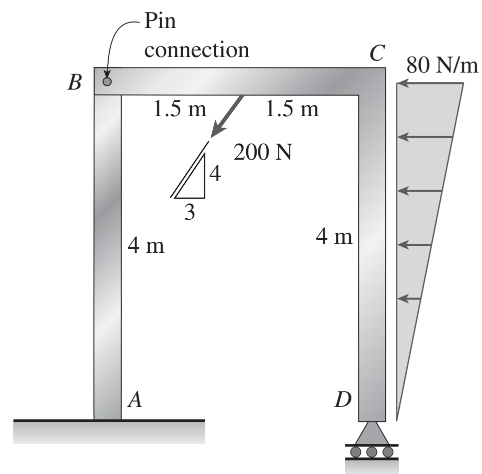
$$V = -A_x - \frac{1}{2}\left(\frac{3}{12}\,90 \text{ lb/ft}\right)3 \text{ ft} = 506 \text{ lb}$$

$$M = -M_A - A_x 3 \text{ ft} - \frac{1}{2}\left(\frac{3}{12}\,90 \text{ lb/ft}\right)3 \text{ ft}\left(\frac{1}{3}3 \text{ ft}\right) = -2734 \text{ lb-ft}$$



Problem 1.2-14 A plane frame is restrained at joints A and D, as shown in the figure. Members AB and BCD are pin connected at B. A triangularly distributed lateral load with peak intensity of 80 N/m acts on CD. An inclined concentrated force of 200 N acts at the mid-span of BC.

(a) Find reactions at supports A and D.
(b) Find resultant forces in the pins at B and C.



Solution 1.2-14

(a) Statics

$\Sigma F_x = 0 \qquad A_x = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$

$\Sigma M_{BRHFB} = 0 \quad D_y = \frac{1}{3\text{ m}}\left[\frac{4}{5}(200\text{ N})(1.5\text{ m}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{1}{3}4\text{ m}\right)\right]$

$= 151.1\text{ N} <$ use right hand FBD (BCD only)

$\Sigma F_y = 0 \qquad A_y = -D_y + \frac{4}{5}(200\text{ N}) = 8.89\text{ N}$

$\Sigma M_A = 0 \qquad M_A = \frac{4}{5}(200\text{ N})(1.5\text{ m}) - \frac{3}{5}(200\text{ N})(4\text{ m}) - D_y\,3\text{ m} - \frac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\frac{2}{3}4\text{ m}\right) = -1120\text{ N}\cdot\text{m}$

(b) Resultant force in pin at B

Left hand FBD (see figure)

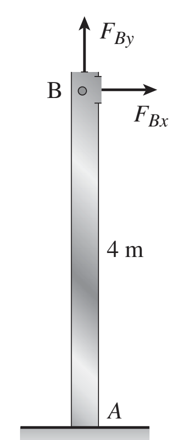
$F_{Bx} = -A_x = -280\text{ N} \qquad F_{By} = -A_y = -8.89\text{ N}$

Right hand FBD

$F_{Bx} = \frac{3}{5}(200\text{ N}) + \frac{1}{2}(80\text{ N/m})\,4\text{ m} = 280\text{ N}$

$F_{By} = \frac{4}{5}(200\text{ N}) - D_y = 8.89\text{ N}$

$\boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 280\text{ N}}$



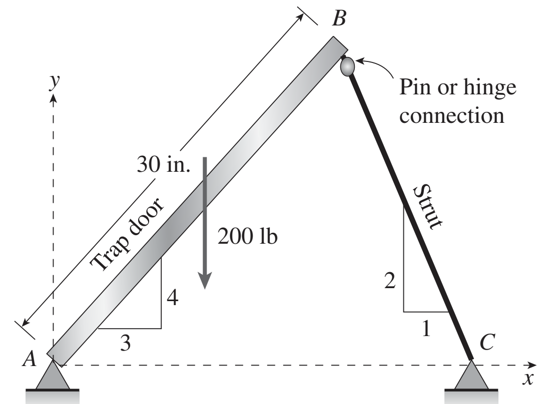
Left hand FBD

Problem 1.2-15 A 200 lb trap door (AB) is supported by a strut (BC) which is pin connected to the door at B (see figure).

(a) Find reactions at supports A and C.
(b) Find internal stress resultants N, V, and M on the trap door at 20 in. from A.

$$L_{BC} = \frac{\frac{4}{5}30\text{ in.}}{\frac{2}{\sqrt{5}}} = 26.833\text{ in.}$$

$$L_{AC} = \frac{3}{5}(30\text{ in.}) + \frac{1}{\sqrt{5}}L_{BC} = 30\text{ in.}$$



Solution 1.2-15

(a) STATICS

$\Sigma M_A = 0 \qquad C_y = \frac{1}{L_{AC}}\left[200\text{ lb}\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)30\text{ in.}\right] = 60\text{ lb} \qquad C_x = \frac{-1}{2}C_y = -30\text{ lb}$

$\Sigma F_x = 0 \qquad A_x = -C_x = 30\text{ lb}$ (resultant of C_x and C_y acts along line of strut)

$\Sigma F_y = 0 \qquad A_y = 200\text{ lb} - C_y = 140\text{ lb}$

(b) INTERNAL STRESS RESULTANTS N, V, M (SEE FIGURE)

Distributed weight of door in $-y$ direction $\qquad w = \frac{200\text{ lb}}{30\text{ in.}} = 6.667\text{ lb/in.}$

Components of w along and perpendicular to door

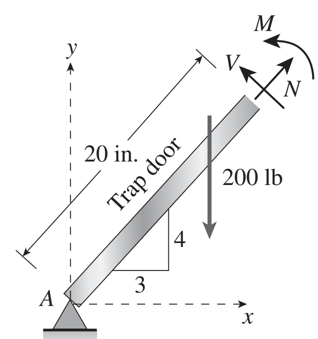
$$w_a = \frac{4}{5}w = 5.333\text{ lb/in.} \qquad w_p = \frac{3}{5}w = 4\text{ lb/in.}$$

$$N = w_a(20\text{ in.}) - \frac{3}{5}A_x - \frac{4}{5}A_y = -23.333\text{ lb}$$

$$V = -w_p(20\text{ in.}) - \frac{4}{5}A_x + \frac{3}{5}A_y = -20\text{ lb}$$

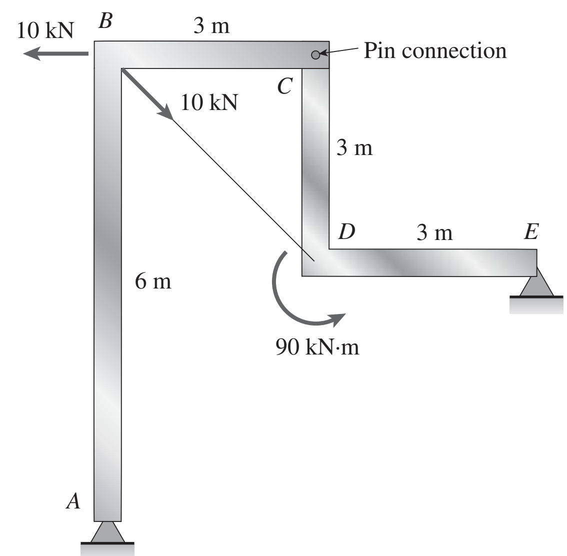
$$M = -w_p(20\text{ in.})\frac{20\text{ in.}}{2} - \frac{4}{5}A_x(20\text{ in.}) + \frac{3}{5}A_y(20\text{ in.}) = 33.333\text{ lb-ft}$$

$\boxed{N = -23.3\text{ lb}} \qquad \boxed{V = -20\text{ lb}} \qquad \boxed{M = 33.3\text{ lb-ft}}$



Problem 1.2-16 A plane frame is constructed by using a pin connection between segments ABC and CDE. The frame has pin supports at A and E and has joint loads at B and D (see figure).

(a) Find reactions at supports A and E.
(b) Find resultant force in the pin at C.



Solution 1.2-16

(a) STATICS

$\Sigma M_A = 0$

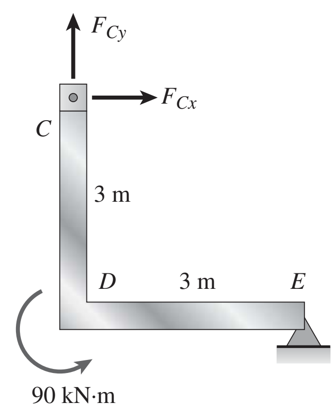
$$10\text{ kN}(6\text{ m}) - 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right)(6\text{ m}) + 90\text{ kN·m} + E_y(6\text{ m}) - E_x 3\text{ m} = 6E_y\text{ m} - 3E_x\text{ m} + 150\text{ kN·m} - 30\sqrt{2}\text{ kN·m}$$

so $\quad 6E_y\text{m} - 3E_x\text{m} + 150\text{ kN·m} - 30\sqrt{2}\text{ kN·m} = 0$

or $\quad -E_x + 2E_y = \dfrac{-(150\text{ kN·m} - 30\sqrt{2}\text{ kN·m})}{3\text{ m}} = -35.858\text{ kN}$

$\Sigma M_{CRHFB} = 0$ < right hand FBD (CDE) - see figure.

$(E_x + E_y)3\text{ m} = -90\text{ kN·m} \qquad E_x + E_y = \dfrac{-90\text{ kN·m}}{3\text{ m}} = -30\text{ kN}$



Solving $\quad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}^{-1}\begin{pmatrix} -35.858\text{ kN} \\ -30\text{ kN} \end{pmatrix} = \begin{pmatrix} -8.05 \\ -21.95 \end{pmatrix}\text{kN}$

$\boxed{E_x = -8.05\text{ kN}} \quad \boxed{E_y = -22\text{ kN}}$

$\Sigma F_x = 0 \qquad A_x = -E_x + 10\text{ kN} - 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 10.98\text{ kN} \qquad \boxed{A_x = 10.98\text{ kN}}$

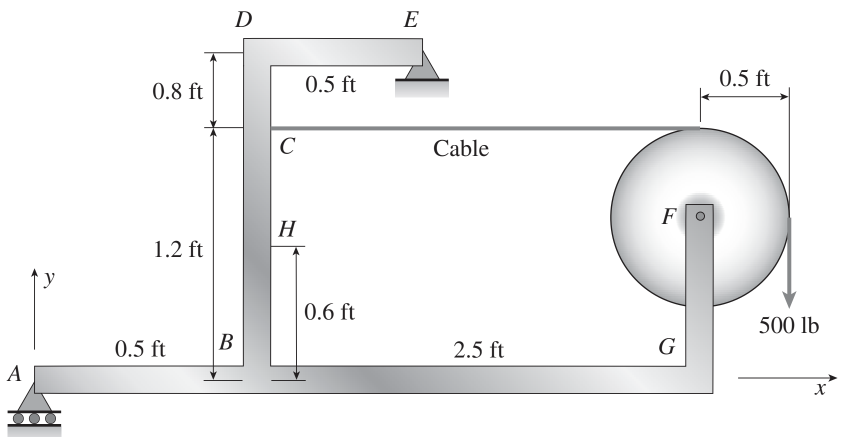
$\Sigma F_y = 0 \qquad A_y = -E_y + 10\text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 29.07\text{ kN} \qquad \boxed{A_y = 29.1\text{ kN}}$

(b) RIGHT HAND FBD $\qquad C_x = -E_x = 8.05\text{ kN} \qquad C_y = -E_y = 22\text{ kN}$

$\boxed{\text{Resultant}_C = \sqrt{C_x^2 + C_y^2} = 23.4\text{ kN}}$

Problem 1.2-17 A plane frame with pin supports at A and E has a cable attached at C, which runs over a frictionless pulley at F (see figure). The cable force is known to be 500 lb.

(a) Find reactions at supports A and E.
(b) Find internal stress resultants, N, V, and M at point H.



Solution 1.2-17

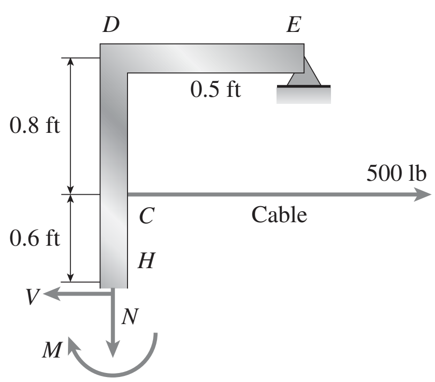
(a) Statics

$$\Sigma F_x = 0 \qquad E_x = 0$$

$$\Sigma M_E = 0 \qquad A_y = \frac{1}{1\text{ ft}}(-500\text{ lb} \times 2.5\text{ ft}) = -1250\text{ lb}$$

$$\Sigma F_y = 0 \qquad E_y = 500\text{ lb} - A_y = 1750\text{ lb}$$

(b) Use upper (see figure below) or lower FBD to find stress resultants N, V, and M at H



$$\Sigma F_x = 0 \qquad V = E_x + 500\text{ lb} = 500\text{ lb}$$

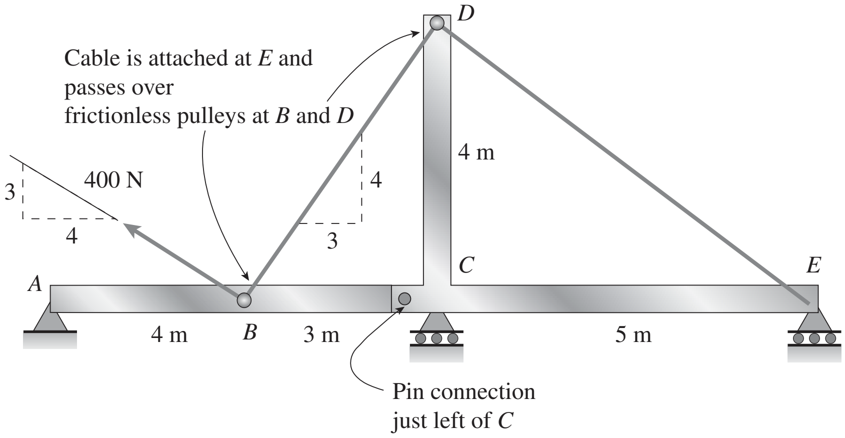
$$\Sigma F_y = 0 \qquad N = E_y = 1750\text{ lb}$$

$$\Sigma M_H = 0$$

$$M = -0.6\text{ ft}(500\text{ lb}) - E_x 1.4\text{ ft} + E_y 0.5\text{ ft} = 575\text{ lb-ft}$$

Problem 1.2-18 A plane frame with a pin support at A and roller supports at C and E has a cable attached at E, which runs over frictionless pulleys at D and B (see figure). The cable force is known to be 400 N. There is a pin connection just to the left of joint C.

(a) Find reactions at supports A, C, and E.
(b) Find internal stress resultants N, V, and M just to the right of joint C.
(c) Find resultant force in the pin near C.



Solution 1.2-18

(a) Statics

$$\Sigma F_x = 0 \qquad A_x = \frac{4}{5}(400\text{ N}) = 320\text{ N} \qquad \boxed{A_x = 320\text{ N}}$$

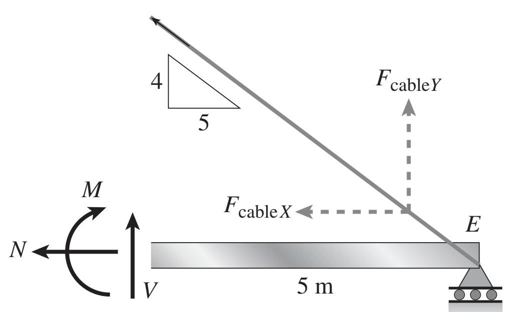
Use left hand FBD (cut through pin just left of C)

$$\Sigma M_C = 0 \qquad A_y = \frac{1}{7\text{ m}}\left[\left[\frac{-3}{5}(400\text{ N}) - \frac{4}{5}(400\text{ N})\right](3\text{ m})\right] = -240\text{ N} \qquad \boxed{A_y = -240\text{ N}}$$

Use entire FBD $\qquad \Sigma M_C = 0 \qquad E_y = \dfrac{1}{5\text{ m}}\left[A_y(7\text{ m}) + \left(\dfrac{3}{5}400\text{ N}\right)(3\text{ m})\right] = -192\text{ N} \qquad \boxed{E_y = -192\text{ N}}$

$$\Sigma F_y = 0 \qquad C_y = -A_y - E_y - \frac{3}{5}(400\text{ N}) = 192\text{ N} \qquad \boxed{C_y = 192\text{ N}}$$

(b) N, V, and M just right of C; use right hand FBD $\qquad F_{cableX} = 400\text{ N}\left(\dfrac{5}{\sqrt{4^2 + 5^2}}\right) = 312.348\text{ N}$



$$F_{cableY} = \frac{4}{5}F_{cableX} = 249.878\text{ N}$$

$$\Sigma F_x = 0 \qquad \boxed{N_x = -F_{cableX} = -312\text{ N}}$$

$$\Sigma F_y = 0 \qquad \boxed{V = -F_{cableY} - E_y = -57.9\text{ N}}$$

$$\Sigma M_C = 0 \qquad M = \left(F_{cableY} + E_y\right)(5\text{ m}) = \boxed{289\text{ N}\cdot\text{m}}$$

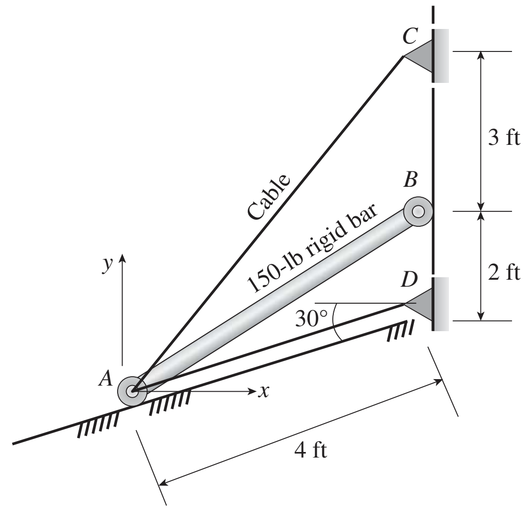
(c) Resultant force in pin just left of C; use left hand FBD $\qquad A_x = 320\text{ N}$

$$F_{Cx} = -A_x + \left(\frac{4}{5} - \frac{3}{5}\right)400\text{ N} = -240\text{ N} \qquad F_{Cy} = -A_y - \left(\frac{3}{5} + \frac{4}{5}\right)400\text{ N} = -320\text{ N}$$

$$\boxed{\text{Res}_C = \sqrt{{F_{Cx}}^2 + {F_{Cy}}^2} = 400\text{ N}}$$

Problem 1.2-19 A 150-lb rigid bar *AB*, with frictionless rollers at each end, is held in the position shown in the figure by a continuous cable *CAD*. The cable is pinned at *C* and *D* and runs over a pulley at *A*.

(a) Find reactions at supports *A* and *B*.
(b) Find the force in the cable.



Solution 1.2-19

(a) Statics $W = 150$ lb

$$\Sigma M_A = 0 \qquad B_x(4) + W\left(\frac{2\sqrt{3}}{2}\right) = 0 \text{ solve, } B_x = -\frac{75\sqrt{3}}{2}$$

$$\text{so} \qquad B_x = -\frac{75\sqrt{3}}{2} = -64.952$$

$$\Sigma F_x = 0 \qquad -A\sin(30°) + B_x + T\cos(30°) + T\cos\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) = 0$$

$$\Sigma F_y = 0 \qquad A\cos(30°) + T\sin(30°) + T\sin\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) = W$$

$$\begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} -\sin(30°) & \cos(30°) + \cos\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) \\ \cos(30°) & \sin(30°) + \sin\left(\arctan\left(\frac{7}{2\sqrt{3}}\right)\right) \end{pmatrix}^{-1} \begin{pmatrix} -B_x \\ W \end{pmatrix} \qquad \begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} 57.713 \\ 71.634 \end{pmatrix} \text{ lb}$$

Support reactions

$\boxed{B_x = -65}$ $\boxed{A = 57.7}$ Units = lbs

$A_x = -A\sin(30°) = -28.9$ lb $\qquad A_y = A\cos(30°) = 50$ lb

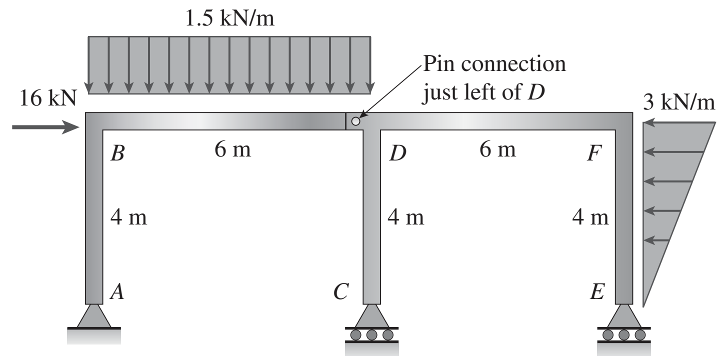
$\sqrt{A_x^2 + A_y^2} = 57.713$

(b) Cable force is T (lbs) from above solution

$\boxed{T = 71.6 \text{ lb}}$

Problem 1.2-20 A plane frame has a pin support at A and roller supports at C and E (see figure). Frame segments ABD and $CDEF$ are joined just left of joint D by a pin connection.

(a) Find reactions at supports A, C, and E.
(b) Find the resultant force in the pin just left of D.



Solution 1.2-20

(a) STATICS

RIGHT-HAND FBD

$$\Sigma M_{\text{pin}} = 0 \qquad E_y = \frac{1}{6\text{ m}}\left[\frac{1}{2}(3\text{ kN/m})\,4\text{ m}\left(\frac{1}{3}4\text{ m}\right)\right] = 1.333\text{ kN} \qquad \boxed{E_y = 1.333\text{ kN}}$$

ENTIRE FBD

$$\Sigma M_A = 0 \quad C_y = \frac{1}{6\text{ m}}\left[-E_y 12\text{ m} + (16\text{ kN})4\text{ m} + (1.5\text{ kN/m})\,6\text{ m}(3\text{ m}) - \frac{1}{2}(3\text{ kN/m})\,4\text{ m}\left(\frac{2}{3}4\text{ m}\right)\right] = 9.833\text{ kN}$$

$$\boxed{C_y = 9.83\text{ kN}}$$

$$\Sigma F_y = 0 \qquad A_y = -C_y - E_y + (1.5\text{ kN/m})\,6\text{ m} = -2.167\text{ kN} \qquad \boxed{A_y = -2.17\text{ kN}}$$

$$\Sigma F_x = 0 \qquad A_x = -16\text{ kN} + \frac{1}{2}(3\text{ kN/m})\;4\text{ m} = -10\text{ kN} \qquad \boxed{A_x = -10\text{ kN}}$$

(b) RESULTANT FORCE IN PIN; USE EITHER RIGHT HAND OR LEFT HAND FBD (CUT THROUGH PIN EXPOSING PIN FORCES F_{Dx} AND F_{Dy}) THEN SUM FORCES IN x AND y DIRECTIONS FOR EITHER FBD

LHFB:

$$F_{Dx} = -16\text{ kN} - A_x = -6\text{ kN}$$

$$F_{Dy} = -A_y + (1.5\text{ kN/m})\,6\text{ m} = 11.167\text{ kN}$$

$$\text{Resultant}_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = 12.68\text{ kN}$$

RHFB:

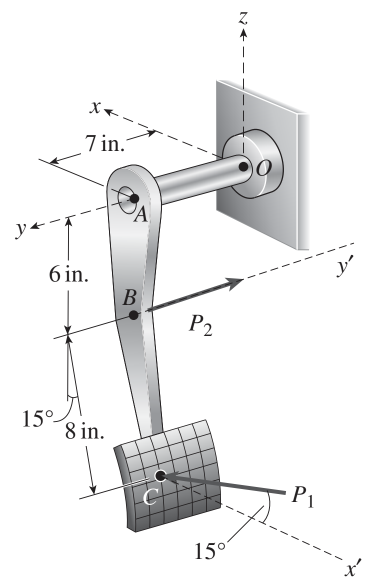
$$F_{Dx} = \frac{1}{2}(3\text{ kN/m})\,4\text{ m} = 6\text{ kN}$$

$$F_{Dy} = -C_y - E_y = -11.167\text{ kN}$$

$$\boxed{\text{Resultant}_D = 12.68\text{ kN}}$$

Problem 1.2-21 A special vehicle brake is clamped at O, (when the brake force P_1 is applied–see figure). Force $P_1 = 50$ lb and lies in a plane which is parallel to the xz plane and is applied at C normal to line BC. Force $P_2 = 40$ lb and is applied at B in the $-y$ direction.

(a) Find reactions at support O.
(b) Find internal stress resultants N, V, T, and M at the midpoint of segment OA.



Solution 1.2-21

(a) STATICS $P_1 = 50$ lb $P_2 = 40$ lb

$\Sigma F_x = 0 \qquad O_x = -P_1\cos(15°) = -48.3 \text{ lb} \qquad \Sigma F_y = 0 \qquad O_y = P_2 = 40 \text{ lb}$

$\Sigma F_z = 0 \qquad O_z = P_1\sin(15°) = 12.94 \text{ lb}$

$\Sigma M_x = 0 \qquad M_{Ox} = P_2 6 \text{ in.} + P_1\sin(15°)(7 \text{ in.}) = 331 \text{ lb-in.}$

$\Sigma M_y = 0 \qquad M_{Oy} = P_1\sin(15°)(8 \text{ in.} \sin(15°)) + P_1\cos(15°)(6 \text{ in.} + 8 \text{ in.} \cos(15°))$

$M_{Oy} = 690 \text{ lb-in.}$

$\Sigma M_z = 0 \qquad M_{Oz} = -P_1\cos(15°)(7 \text{ in.}) = -338 \text{ lb-in.}$

(b) INTERNAL STRESS RESULTANTS AT MIDPOINT OF OA

$N = -O_y = -40 \text{ lb}$

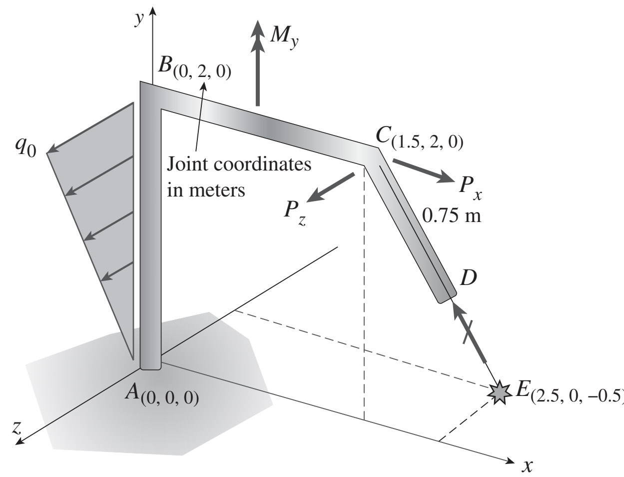
$V_x = -O_x = 48.3 \text{ lb} \qquad V_z = -O_z = -12.94 \text{ lb} \qquad V = \sqrt{V_x^2 + V_z^2} = 50 \text{ lb}$

$T = -M_{Oy} = -690 \text{ lb-in.}$

$M_x = -M_{Ox} = -330.59 \text{ lb-in.} \qquad M_z = -M_{Oz} = 338.07 \text{ lb-in.} \qquad M = \sqrt{M_x^2 + M_z^2} = 473 \text{ lb-in.}$

Problem 1.2-22 Space frame $ABCD$ is clamped at A, except it is free to translate in the x-direction. There is also a roller support at D, which is normal to line CDE. A triangularly distributed force with peak intensity $q_0 = 75$ N/m acts along AB in the positive z direction. Forces $P_x = 60$ N and $P_z = -45$ N are applied at joint C and a concentrated moment $M_y = 120$ N·m acts at the mid-span of member BC.

(a) Find reactions at supports A and D.
(b) Find internal stress resultants N, V, T, and M at the mid-height of segment AB.



Solution 1.2-22

FORCES

$P_x = 60$ N $\quad P_z = -45$ N $\quad M_y = 120$ N·m $\quad q_0 = 75$ N/m

$$F_C = \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} 60 \\ 0 \\ -45 \end{pmatrix} \text{N} \qquad R_A = \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix}$$

VECTOR ALONG MEMBER CD

$$r_{EC} = \begin{bmatrix} 1.5 - 2.5 \\ 2 - 0 \\ 0 - (-0.5) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0.5 \end{bmatrix} \qquad |r_{EC}| = 2.291 \qquad e_{EC} = \frac{r_{EC}}{|r_{EC}|} = \begin{pmatrix} -0.436 \\ 0.873 \\ 0.218 \end{pmatrix}$$

(a) STATICS (FORCE AND MOMENT EQUILIBRIUM)

$$\Sigma F = 0 \qquad \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} + \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = 0 \qquad \text{resultant of triangular load:} \quad R_T = \frac{1}{2} q_0 (2 \text{ m}) = 75 \text{ N}$$

$$\text{where} \qquad \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = D e_{EC}$$

SOLVING ABOVE THREE EQUATIONS:

$$\Sigma F_x = 0 \qquad D_x = -P_x \text{ so} \qquad D = \frac{-P_x}{e_{EC_1}} \qquad D = 137.477 \text{ N} \qquad \boxed{D_x = -60 \text{ N}}$$

$$\Sigma F_y = 0 \qquad D_y = e_{EC_2} D \qquad \boxed{D_y = 120 \text{ N}} \qquad \boxed{A_y = -D_y = -120 \text{ N}}$$

$$\Sigma F_z = 0 \qquad D_z = e_{EC_3} D \qquad \boxed{D_z = 30 \text{ N}} \qquad \sqrt{D_x^2 + D_y^2 + D_z^2} = 137.477 \text{ N}$$

$$\text{so} \quad A_z = -D_z - R_T - P_z \qquad \boxed{A_z = -60 \text{ N}}$$

$$\Sigma M_A = 0$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} + r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = 0$$

$$r_{AE} = \begin{pmatrix} 2.5 - 0 \\ 0 - 0 \\ -0.5 - 0 \end{pmatrix}\text{m} \qquad D = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \qquad D = \begin{pmatrix} -60 \\ 120 \\ 30 \end{pmatrix}\text{N} \qquad |D| = 137.477\ \text{N} \qquad r_{AE} \times D = \begin{pmatrix} 60 \\ -45 \\ 300 \end{pmatrix}\text{N}\cdot\text{m}$$

$$r_{AC} = \begin{pmatrix} 1.5 - 0 \\ 2 - 0 \\ 0 - 0 \end{pmatrix}\text{m} \qquad r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} -90 \\ 67.5 \\ -120 \end{pmatrix}\text{J} \qquad r_{cg} = \begin{pmatrix} 0 \\ \frac{2}{3}(2\ \text{m}) \\ 0 \end{pmatrix} \qquad r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix}\text{N}\cdot\text{m}$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = -\left[r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} \right] = \begin{pmatrix} -70 \\ -142.5 \\ -180 \end{pmatrix}\text{N}\cdot\text{m} \qquad \boxed{\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} -70 \\ -142.5 \\ -80 \end{pmatrix}\text{N}\cdot\text{m}}$$

(b) RESULTANTS AT MID-HEIGHT OF AB (SEE FBD IN FIGURE BELOW)

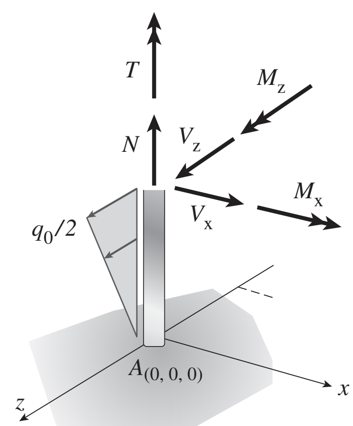
$$\boxed{N = -A_y = 120\ \text{N}} \qquad V_x = -D_x - P_x = 0\ \text{N} \quad V_z = -A_z - \frac{1}{2}\frac{q_0}{2}(2\,\text{m})/2 = 41.25\ \text{N} \quad \boxed{V = V_z = 41.3\ \text{N}}$$

$$\boxed{T = -M_{Ay} = 142.5\ \text{N}\cdot\text{m}} \quad M_x = -M_{Ax} + A_z(1\ \text{m}) + \frac{1}{2}\frac{q_0}{2}1\ \text{m}\left(\frac{1}{3}1\ \text{m}\right) = 16.25\ \text{N}\cdot\text{m}$$

$$M_z = -M_{Az} = 180\ \text{N}\cdot\text{m}$$

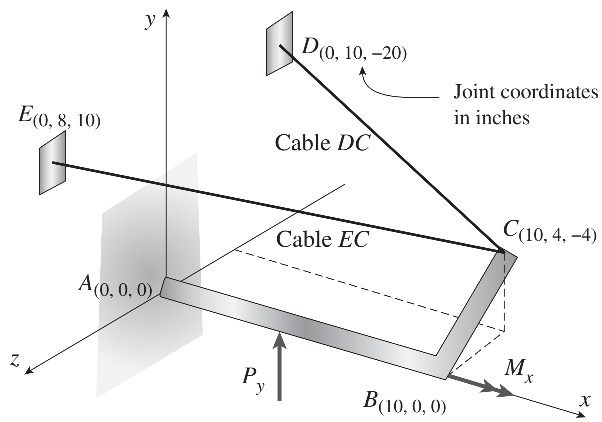
$$M_{\text{resultant}} = \sqrt{M_x^2 + M_z^2} = 180.732\ \text{N}\cdot\text{m}$$

$$\boxed{M_{\text{resultant}} = 180.7\ \text{N}\cdot\text{m}}$$



Problem 1.2-23 Space frame ABC is clamped at A except it is free to rotate at A about the x and y axes. Cables DC and EC support the frame at C. Forces $P_Y = -50$ lb is applied at mid-span of AB and a concentrated moment $M_X = -20$ in-lb acts at joint B.

(a) Find reactions at supports A.
(b) Find cable tension forces.



Solution 1.2-23

POSITION AND UNIT VECTORS

$$r_{AB} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} \qquad r_{AP} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \qquad r_{AC} = \begin{pmatrix} 10 \\ 4 \\ -4 \end{pmatrix} \qquad r_{CD} = \begin{bmatrix} 0 - 10 \\ 10 - 4 \\ -20 - (-4) \end{bmatrix} = \begin{pmatrix} -10 \\ 6 \\ -16 \end{pmatrix} \qquad e_{CD} = \frac{r_{CD}}{|r_{CD}|} = \begin{pmatrix} -0.505 \\ 0.303 \\ -0.808 \end{pmatrix}$$

APPLIED FORCE AND MOMENT

$$r_{CE} = \begin{bmatrix} 0 - 10 \\ 8 - 4 \\ 10 - (-4) \end{bmatrix} = \begin{pmatrix} -10 \\ 4 \\ 14 \end{pmatrix} \qquad e_{CE} = \frac{r_{CE}}{|r_{CE}|} = \begin{pmatrix} -0.566 \\ 0.226 \\ 0.793 \end{pmatrix}$$

$$P_y = -50 \text{ lb} \qquad M_x = -20 \text{ lb-in.}$$

STATICS FORCE AND MOMENT EQUILIBRIUM

First sum moment about point A

$$\Sigma M_A = 0$$

$$M_A = \begin{pmatrix} 0 \\ 0 \\ M_{Az} \end{pmatrix} + r_{AP} \times \begin{pmatrix} 0 \\ P_y \\ 0 \end{pmatrix} + \begin{pmatrix} M_x \\ 0 \\ 0 \end{pmatrix} + r_{AC} \times \left(T_D\, e_{CD} + T_E\, e_{CE}\right) = \begin{pmatrix} -2.0203\, T_D + 4.0762\, T_E - 20.0 \\ 10.102\, T_D + -5.6614\, T_E \\ M_{Az} + 5.0508\, T_D + 4.5291\, T_E - 250.0 \end{pmatrix}$$

Solve moment equilibrium equations for moments about x and y axes to get cable tension forces

$$\begin{pmatrix} T_D \\ T_E \end{pmatrix} = \begin{pmatrix} -2.0203 & 4.0762 \\ 10.102 & -5.6614 \end{pmatrix}^{-1} \begin{pmatrix} 20 \\ 0 \end{pmatrix} = \begin{pmatrix} 3.81 \\ 6.79 \end{pmatrix} \text{lb} \qquad \text{(b)}$$

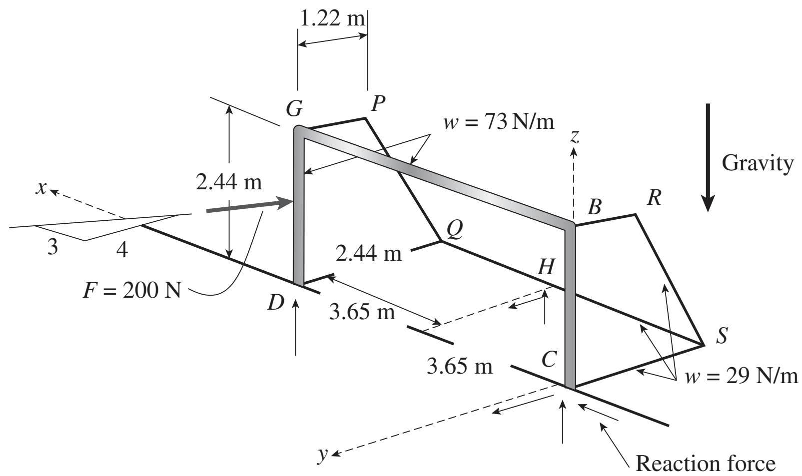
Next, solve moment equilibrium equation about z axis now that cable forces are known

$$M_{Az} = -(5.0508\, T_D + 4.5291\, T_E - 250.0) = 200 \text{ lb-in.} \qquad \text{(a)}$$

Finally, use force equilibrium to find reaction forces at point A

$$\Sigma F = 0 \qquad \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = -\begin{pmatrix} 0 \\ P_y \\ 0 \end{pmatrix} - (T_D\, e_{CD} + T_E\, e_{CE}) = \begin{pmatrix} 5.77 \\ 47.31 \\ -2.31 \end{pmatrix} \text{lb}$$

Problem 1.2-24 A soccer goal is subjected to gravity loads (in the $-z$ direction, $w = 73$ N/m for DG, BG, and BC; $w = 29$ N/m for all other members; see figure) and a force $F = 200$ N applied eccentrically at mid-height of member DG. Find reactions at supports C, D, and H.



Solution 1.2-24

FIND MEMBER LENGTHS

$$L_{QS} = 2(3.65\text{ m}) = 7.3\text{ m} \qquad L_{RS} = \sqrt{(2.44\text{ m})^2 + (2.44\text{ m} - 1.22\text{ m})^2} = 2.728\text{ m} \quad L_{PQ} = L_{RS}$$

Assume that soccer goal is supported only at points C, H, and D (see reaction force components at each location in figure)

STATICS SUM MOMENT ABOUT EACH AXIS AND FORCES IN EACH AXIS DIRECTION $F = 200$ N

$\Sigma M_x = 0$ TO FIND REACTION COMPONENT H_y:

Find moments about x due to for component F_y and also for distributed weight of each frame component

$$M_{xGP} = \frac{(1.22\text{ m})^2}{2}(29\text{ N/m}) \qquad M_{xBR} = M_{xGP} \qquad M_{xDQ} = \frac{(2.44\text{ m})^2}{2}(29\text{ N/m}) \qquad M_{xCS} = M_{xDQ}$$

$$M_{xRS} = L_{RS}(29\text{ N/m})\left(1.22\text{ m} + \frac{1.22\text{ m}}{2}\right) \qquad M_{xPQ} = M_{xRS} \qquad M_{xQS} = L_{QS}(29\text{ N/m})(2.44\text{ m})$$

$$H_z = \frac{1}{2.44\text{ m}}\left[\frac{4}{5}F\left(\frac{2.44\text{ m}}{2}\right) + 2M_{xGP} + 2M_{xDQ} + 2M_{xPQ} + M_{xQS}\right] = 498.818\text{ N} \qquad \boxed{H_z = 499\text{ N}}$$

$\Sigma M_y = 0$ TO FIND REACTION FORCE D_z:

$$M_{yGD} = 2.44\text{ m}(73\text{ N/m})L_{QS} \qquad M_{yGP} = 1.22\text{ m}(29\text{ N/m})L_{QS} \qquad M_{yDQ} = 2.44\text{ m}(29\text{ N/m})L_{QS}$$

$$M_{yPQ} = L_{RS}(29\text{ N/m})L_{QS} \qquad M_{yBG} = L_{QS}(73\text{ N/m})\frac{L_{QS}}{2} \qquad M_{yQS} = L_{QS}(29\text{ N/m})\frac{L_{QS}}{2}$$

$$D_z = \frac{1}{L_{QS}}\left[M_{yGD} + M_{yGP} + M_{yDQ} + M_{yPQ} + M_{yBG} + M_{yQS} - H_z\frac{L_{QS}}{2} - \frac{3}{5}F\left(\frac{2.44\text{ m}}{2}\right)\right] = 466.208\text{ N}$$

$$\boxed{D_z = 466\text{ N}}$$

$\Sigma M_z = 0$ TO FIND REACTION FORCE H_y:

$$H_y = \frac{1}{3.65\text{ m}}\left(\frac{4}{5}FL_{QS}\right) = 320\text{ N} \qquad \boxed{H_y = 320\text{ N}}$$

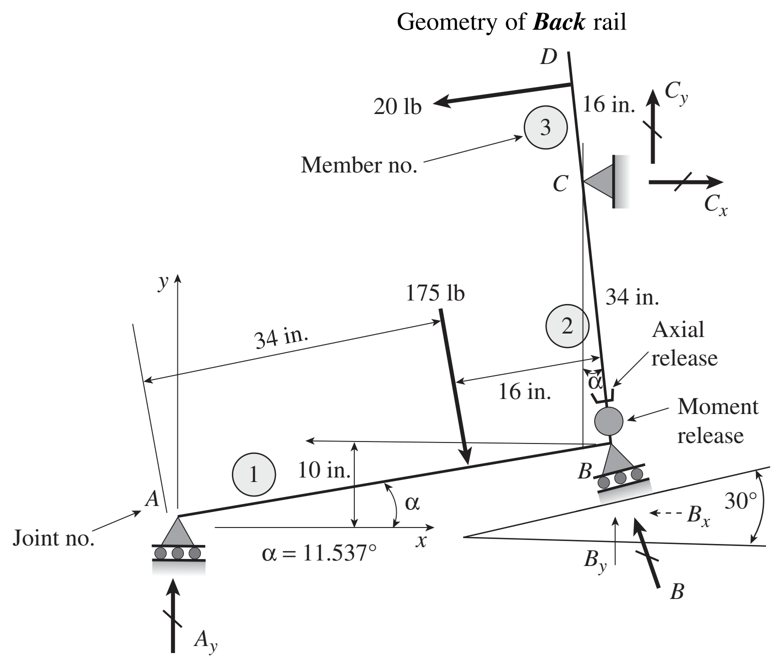
$\Sigma F_x = 0$ TO FIND REACTION FORCE C_x: $\boxed{C_x = \frac{3}{5}F = 120\text{ N}}$

$\Sigma F_y = 0$ TO FIND REACTION FORCE C_y: $C_y = -H_y + \frac{4}{5}F = -160\text{ N}$ $\boxed{C_y = -160\text{ N}}$

$\Sigma F_z = 0$ TO FIND REACTION FORCE C_z:

$$C_z = -D_z - H_z + (29\text{ N/m})(21.22\text{ m} + 22.44\text{ m} + 2L_{RS} + L_{QS})$$
$$+ (73\text{ N/m})(22.44\text{ m} + L_{QS}) = 506.318\text{ N} \quad \boxed{C_z = 506\text{ N}}$$

Problem 1.2-25 An elliptical exerciser machine (see figure part a) is composed of front and back rails. A simplified plane frame model of the back rail is shown in figure part b. Analyze the plane frame model to find reaction forces at supports A, B, and C for the position and applied loads given in figure part b. Note that there are axial and moment releases at the base of member 2 so that member 2 can lengthen and shorten as the roller support at B moves along the 30° incline. (These releases indicate that the internal axial force N and moment M must be zero at this location.)



Solution 1.2-25

$$\alpha = \arcsin\left(\frac{10}{50}\right) = 11.537°$$ Analysis pertains to this position of exerciser only

STATICS UFBD (CUT AT AXIAL AND MOMENT RELEASES JUST ABOVE B)

Inclined vertical component of reaction at $C = 0$ (due to axial release)

Sum moments about moment release to get inclined normal reaction at C

$$C = \frac{20\text{ lb}(34\text{ in.} + 16\text{ in.})}{34\text{ in.}} = 29.412\text{ lb} \quad \boxed{C_x = C\cos(\alpha) = 28.8\text{ lb}}$$

$$\boxed{C_y = C\sin(\alpha) = 5.88\text{ lb}} \quad \sqrt{C_x^2 + C_y^2} = 29.412\text{ lb}$$

STATICS LFBD (CUT THROUGH AXIAL AND MOMENT RELEASES)

Sum moments to find reaction A_y $\boxed{A_y = \frac{175\text{ lb}(16\text{ in.})}{(34\text{ in.} + 16\text{ in.})\cos(\alpha)} = 57.2\text{ lb}}$

STATICS SUM FORCES FOR ENTIRE FBD TO FIND REACTION AT B

Sum forces in x-direction: $B_x = C_x + 175\,\text{lb}(\sin(\alpha)) - 20\,\text{lb}(\cos(\alpha)) = 44.2\,\text{lb}$ < acts leftward

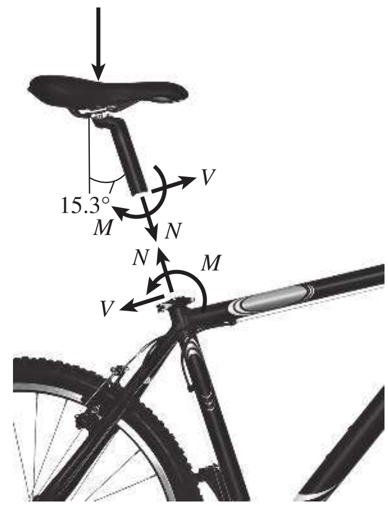
Sum forces in y-direction: $B_y = -A_y - C_y + 175\text{ lb}(\cos(\alpha)) + 20\text{ lb}(\sin(\alpha)) = 112.4\text{ lb}$

$\boxed{B_x = 44.2\text{ lb}}$ $\boxed{B_y = 112.4\text{ lb}}$

Resultant reaction force at B: $B = \sqrt{B_x^2 + B_y^2} = 120.8\text{ lb}$

Problem 1.2-26 A mountain bike is moving along a flat path at constant velocity. At some instant, the rider (weight = 670 N) applies pedal and hand forces, as shown in the figure part a.

(a) Find reactions forces at the front and rear hubs. (Assume that the bike is pin supported at the rear hub and roller supported at the front hub).
(b) Find internal stress resultants N, V, and M in the inclined seat post (see figure part b).



Solution 1.2-26

(a) REACTIONS: SUM MOMENTS ABOUT REAR HUB TO FIND VERTICAL REACTION AT FRONT HUB (FIG. 1)

$$\Sigma M_B = 0$$

$$V_F = \frac{1}{1130}[670(241) - 90(\cos(5°))254 + 200\cos(15°)660 + 2(45)\cos(30°)1021 + 2(45)\sin(30°)752]$$

$$V_F = 335.945\text{ N}$$

Sum forces to get force components at rear hub

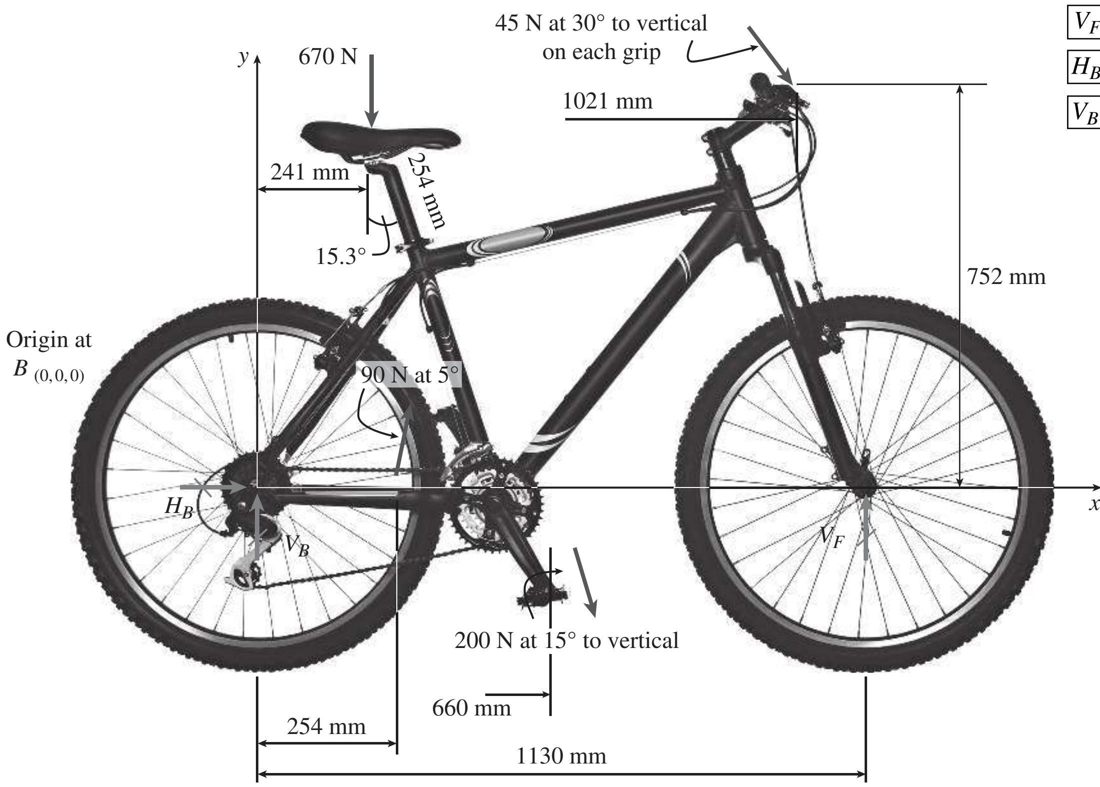
$\Sigma F_{\text{vert}} = 0 \qquad V_B = 670 - 90\cos(5°) + 200\cos(15°) + 2(45)\cos(30°) - V_F = 515.525\text{ N}$

$\Sigma F_{\text{horiz}} = 0 \qquad H_B = -90\sin(5°) - 200\sin(15°) - 2(45)\sin(30°) = -104.608\text{ N}$

$\boxed{V_F = 336\text{ N}}$

$\boxed{H_B = -104.6\text{ N}}$

$\boxed{V_B = 516\text{ N}}$



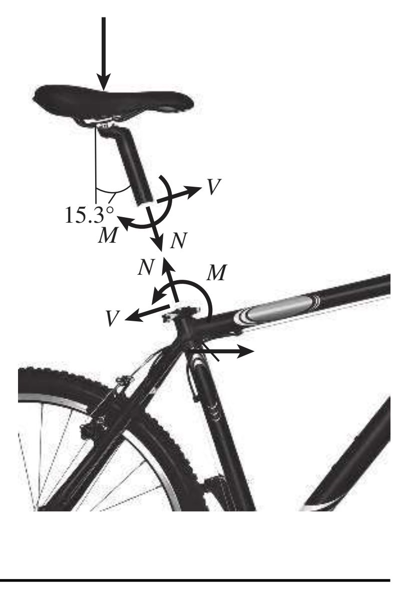
(b) STRESS RESULTANTS N, V, AND M IN SEAT POST (Fig. 2)

SEAT POST RESULTANTS (FIG. 2)

$N = -670\cos(15.3°) = -646.253\text{ N} \qquad \boxed{N = -646\text{ N}}$

$V = 670\sin(15.3°) = 176.795\text{ N} \qquad \boxed{V = 176.8\text{ N}}$

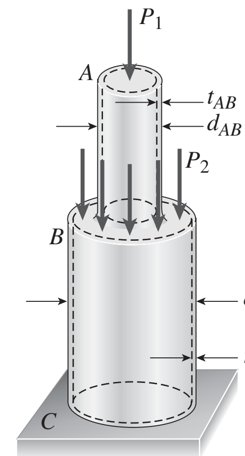
$M = 670\sin(15.3°)\ 254 = 44{,}905.916\text{ N·mm} \qquad \boxed{M = 44.9\text{ N·m}}$



Normal Stress and Strain

Problem 1.3-1 A hollow circular post *ABC* (see figure) supports a load $P_1 = 1700$ lb acting at the top. A second load P_2 is uniformly distributed around the cap plate at *B*. The diameters and thicknesses of the upper and lower parts of the post are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{BC} = 0.375$ in., respectively.

(a) Calculate the normal stress σ_{AB} in the upper part of the post.
(b) If it is desired that the lower part of the post have the same compressive stress as the upper part, what should be the magnitude of the load P_2?
(c) If P_1 remains at 1700 lb and P_2 is now set at 2260 lb, what new thickness of *BC* will result in the same compressive stress in both parts?



Solution 1.3-1

Part (a)

$P_1 = 1700$ lb $\quad d_{AB} = 1.25$ in. $\quad t_{AB} = 0.5$ in.

$d_{BC} = 2.25$ in. $\quad t_{BC} = 0.375$ in.

$$A_{AB} = \frac{\pi[\,d_{AB}{}^2 - (d_{AB} - 2\,t_{AB})^2]}{4}$$

$$A_{AB} = 1.178 \text{ in.}^2 \qquad \sigma_{AB} = \frac{P_1}{A_{AB}}$$

$$\sigma_{AB} = 1443 \text{ psi} \quad \leftarrow$$

Part (b)

$$A_{BC} = \frac{\pi[d_{BC}{}^2 - (d_{BC} - 2t_{BC})^2]}{4}$$

$$A_{BC} = 2.209 \text{ in.}^2 \qquad P_2 = \sigma_{AB}A_{BC} - P_1$$

$$P_2 = 1488 \text{ lbs} \quad \leftarrow$$

CHECK: $$\frac{P_1 + P_2}{A_{BC}} = 1443 \text{ psi}$$

Part (c)

$$P_2 = 2260 \qquad \frac{P_1 + P_2}{\sigma_{AB}} = A_{BC}$$

$$\frac{P_1 + P_2}{\sigma_{AB}} = 2.744$$

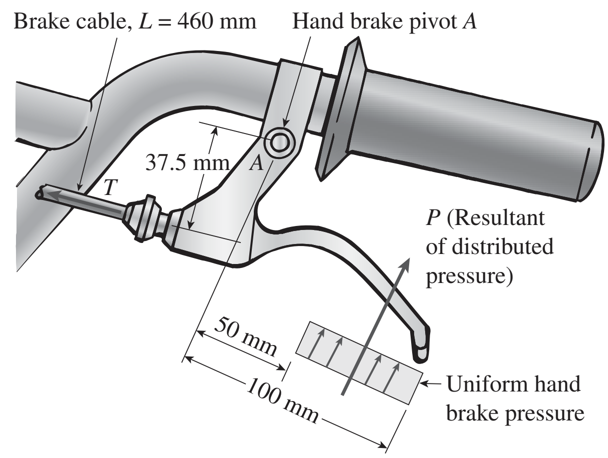
$$(d_{BC} - 2t_{BC})^2 = d_{BC}{}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)$$

$$d_{BC} - 2t_{BC} = \sqrt{d_{BC}{}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)}$$

$$t_{BC} = \frac{d_{BC} - \sqrt{d_{BC}{}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)}}{2}$$

$$t_{BC} = 0.499 \text{ in.} \quad \leftarrow$$

Problem 1.3-2 A force P of 70 N is applied by a rider to the front hand brake of a bicycle (P is the resultant of an evenly distributed pressure). As the hand brake pivots at A, a tension T develops in the 460-mm long brake cable ($A_e = 1.075\ \text{mm}^2$) which elongates by $\delta = 0.214$ mm. Find normal stress σ and strain ε in the brake cable.



Solution 1.3-2

$P = 70$ N $\qquad A_e = 1.075\ \text{mm}^2$

$L = 460$ mm $\qquad \delta = 0.214$ mm

Statics: sum moments about A to get $T = 2P$

$$\sigma = \frac{T}{A_e} \qquad \sigma = 103.2\ \text{MPa} \quad \longleftarrow$$

$$\varepsilon = \frac{\delta}{L} \qquad \varepsilon = 4.65 \times 10^{-4} \quad \longleftarrow$$

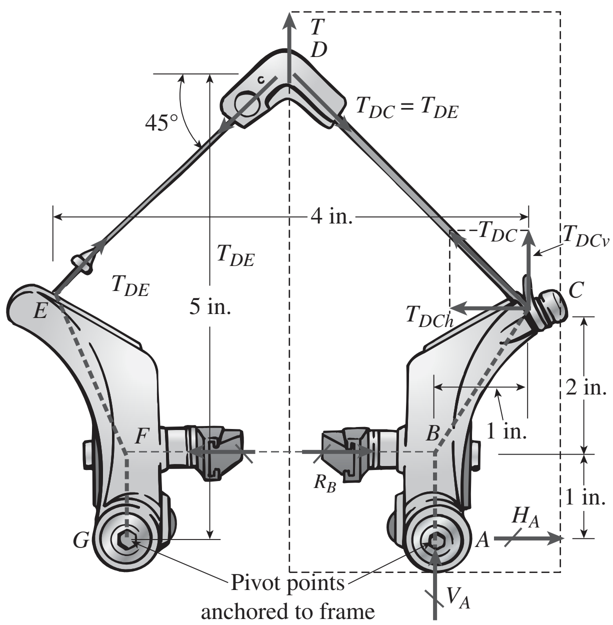
$$E = \frac{\sigma}{\varepsilon} = 1.4 \times 10^5\ \text{MPa}$$

NOTE: (E for cables is approximately 140 GPa)

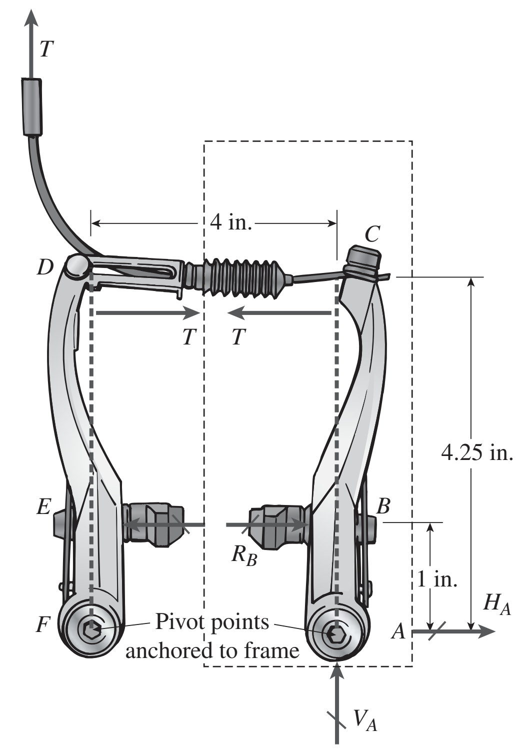
Problem 1.3-3 A bicycle rider would like to compare the effectiveness of cantilever hand brakes [see figure part (a)] versus V-brakes [figure part (b)].

(a) Calculate the braking force R_B at the wheel rims for each of the bicycle brake systems shown. Assume that all forces act in the plane of the figure and that cable tension $T = 45$ lbs. Also, what is the average compressive normal stress σ_c on the brake pad ($A = 0.625$ in.^{2})?

(b) For each braking system, what is the stress in the brake cable (assume effective cross-sectional area of 0.00167 in.^{2})? (*HINT:* Because of symmetry, you only need to use the right half of each figure in your analysis.)



(a) Cantilever brakes



(b) V-brakes

Solution 1.3-3

$T = 45$ lbs $\qquad A_{\text{pad}} = 0.625$ in.^{2}

$A_{\text{cable}} = 0.00167$ in.^{2}

(a) CANTILEVER BRAKES—BRAKING FORCE R_B and PAD PRESSURE

STATICS SUM FORCES AT D TO GET $T_{DCv} = T/2$

$$\sum M_A = 0$$

$R_B(1) = T_{DCh}(3) + T_{DCv}(1)$s

$T_{DCh} = T_{DCv} \qquad T_{DCh} = T/2$

$R_B = 2T \qquad R_B = 90$ lbs $\leftarrow$

so $R_B = 2T$ versus $4.25T$ for V-brakes (next)

$$\sigma_{pad} = \frac{R_B}{A_{pad}} \qquad \sigma_{pad} = 144 \text{ psi} \quad \leftarrow \qquad \frac{4.25}{2} = 2.125$$

$$\sigma_{cable} = \frac{T}{A_{cable}} \qquad \sigma_{cable} = 26{,}946 \text{ psi} \quad \leftarrow \qquad \text{(same for V-brakes (below))}$$

(b) V-BRAKES—BRAKING FORCE R_B AND PAD PRESSURE

$$\sum M_A = 0 \qquad R_B = 4.25T \qquad R_B = 191.3 \text{ lbs} \quad \leftarrow$$

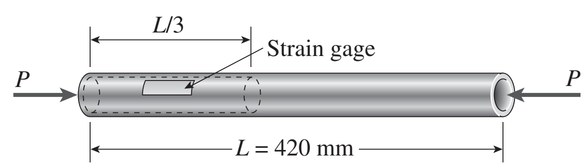
$$\sigma_{pad} = \frac{R_B}{A_{pad}} \qquad \sigma_{pad} = 306 \text{ psi} \quad \leftarrow$$

Problem 1.3-4 A circular aluminum tube of length $L = 420$ mm is loaded in compression by forces P (see figure). The hollow segment of length $L/3$ has outside and inside diameters of 60 mm and 35 mm, respectively. The solid segment of length $2L/3$ has diameter of 60 mm. A strain gage is placed on the outside of the hollow segment of the bar to measure normal strains in the longitudinal direction.

(a) If the measured strain in the hollow segment is $\varepsilon_h = 470 \times 10^{-6}$, what is the strain ε_s in the solid part? (*Hint*: The strain in the solid segment is equal to that in the hollow segment multiplied by the ratio of the area of the hollow to that of the solid segment).

(b) What is the overall shortening δ of the bar?

(c) If the compressive stress in the bar cannot exceed 48 MPa, what is the maximum permissible value of load P?



Solution 1.3-4

$L = 420$ mm $\quad d_2 = 60$ mm $\quad d_1 = 35$ mm $\quad \varepsilon_h = 470(10^{-6}) \quad \sigma_a = 48$ MPa

PART (a)

$$A_s = \frac{\pi}{4}d_2^{\,2} = 2.827 \times 10^{-3}\text{m}^2 \qquad A_h = \frac{\pi}{4}\left(d_2^{\,2} - d_1^{\,2}\right) = 1.865 \times 10^{-3}\text{m}^2$$

$$\varepsilon_h = \frac{A_h}{A_s}\varepsilon_h = 3.101 \times 10^{-4}$$

PART (b)

$$\delta = \varepsilon_h \frac{L}{3} + \varepsilon_s\left(\frac{2L}{3}\right) = 0.1526 \text{ mm} \qquad \varepsilon_h \frac{L}{3} = 0.066 \text{ mm} \qquad \varepsilon_s\left(\frac{2L}{3}\right) = 0.087 \text{ mm}$$

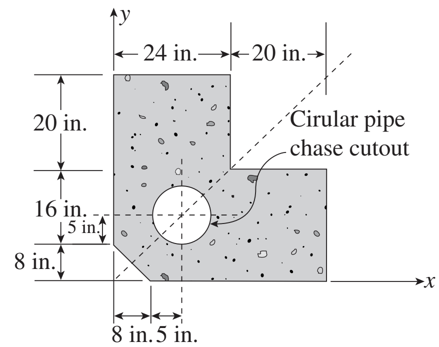
PART (c)

$$P_{maxh} = \sigma_a A_h = 89.535 \text{ kN} \qquad P_{maxs} = \sigma_a A_s = 135.717 \text{ kN} \qquad < \text{lesser value controls}$$

$$\boxed{P_{max} = P_{maxh} = 89.5 \text{ kN}}$$

Problem 1.3-5 The cross section of a concrete corner column that is loaded uniformly in compression is shown in the figure. A circular pipe chase cut-out of 10 in. in diameter runs the height of the column (see figure).

(a) Determine the average compressive stress σ_c in the concrete if the load is equal to 3500 kips.
(b) Determine the coordinates x_c and y_c of the point where the resultant load must act in order to produce uniform normal stress in the column.



Solution 1.3-5

$P = 3500$ kips

$$A = (24 + 20)(20 + 16 + 8) - \left(\frac{1}{2}8^2\right) - 20^2 - \frac{\pi}{4}10^2$$

$A = 1425.46$ in.^{2}

(a) AVERAGE COMPRESSIVE STRESS

$$\sigma_c = \frac{P}{A} \qquad \boxed{\sigma_c = 2.46 \text{ ksi}}$$

(b) CENTROID

$$x_c = \frac{(24+20)^2\,\dfrac{(24+20)}{2} - \left(20^2\right)(24+10) - \dfrac{1}{2}8^2\left(\dfrac{8}{3}\right)}{A} - \frac{\left(\dfrac{\pi}{4}10^2\right)(8+5)}{A}$$

$$\boxed{x_c = 19.56 \text{ in.}}$$

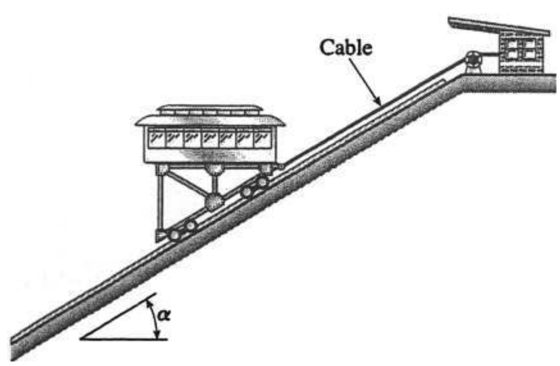
$$y_c = \frac{(24+20)^2\,\dfrac{(24+20)}{2} - \left(20^2\right)(24+10) - \dfrac{1}{2}8^2\left(\dfrac{8}{3}\right)}{A} - \frac{\left(\dfrac{\pi}{4}10^2\right)(8+5)}{A}$$

$$\boxed{y_c = 19.56 \text{ in.}}$$

ˆ x_c and y_c are the same as expected due to symmetry about a diagonal

Problem 1.3-6 A car weighing 130 kN when fully loaded is pulled slowly up a steep inclined track by a steel cable (see figure). The cable has an effective cross-sectional area of 490 mm^2, and the angle α of the incline is 30°.

(a) Calculate the tensile stress σ_t in the cable.
(b) If the allowable stress in the cable is 150 MPa, what is the maximum acceptable angle of the incline for a fully loaded car?



Solution 1.3-6

$W = 130$ kN $\quad \alpha = 30°\quad A = 490\ \mathrm{mm}^2 \quad \sigma_a = 150$ MPa

PART (a)

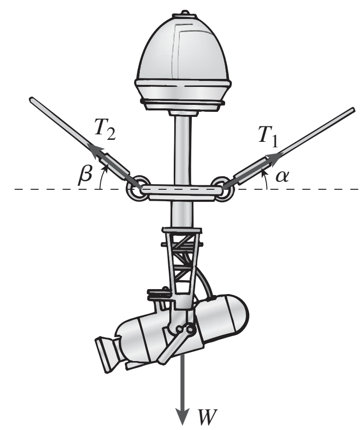
$$\sigma_t = \frac{W\sin(\alpha)}{A} = 132.7 \text{ MPa}$$

PART (b)

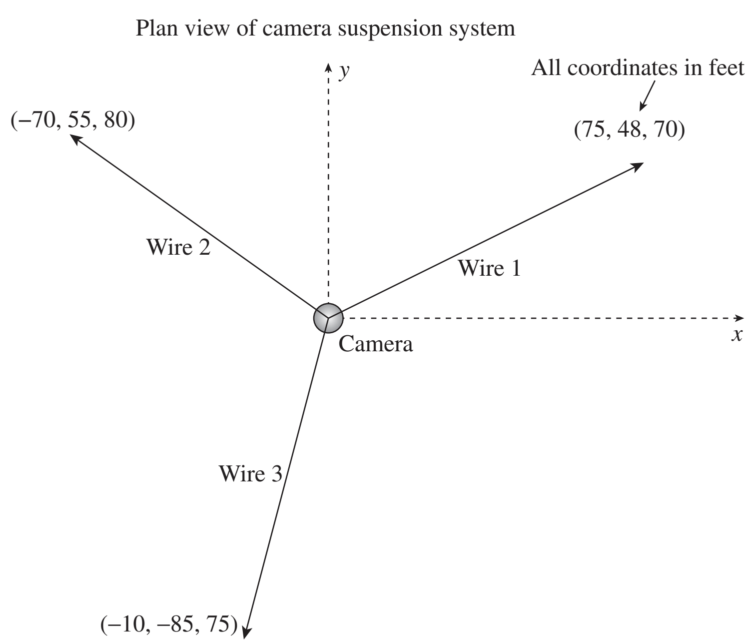
$$\alpha_{max} = \arcsin\left(\frac{\sigma_a A}{W}\right) = 34.4°$$

Problem 1.3-7 Two steel wires support a moveable overhead camera weighing $W = 28$ lb (see figure part a) used for close-up viewing of field action at sporting events. At some instant, wire 1 is at an angle $\alpha = 22°$ to the horizontal and wire 2 is at an angle $\beta = 40°$. Wires 1 and 2 have diameters of 30 and 35 mils, respectively. (Wire diameters are often expressed in mils; one mil equals 0.001 in.)

(a) Determine the tensile stresses σ_1 and σ_2 in the two wires.
(b) If the stresses in wires 1 and 2 must be the same, what is the required diameter of wire 1?
(c) Now, to stabilize the camera for windy outdoor conditions, a third wire is added (see figure part b). Assume the three wires meet at a common point (coordinates (0, 0, 0) above the camera at the instant shown in figure part b). Wire 1 is attached to a support at coordinates (75 ft, 48 ft, 70 ft). Wire 2 is supported at (−70 ft, 55 ft, 80 ft). Wire 3 is supported at (−10 ft, −85 ft, 75 ft). Assume that all three wires have a diameter of 30 mils. Find the tensile stresses in wires 1 to 3.



(a)



(b)

Solution 1.3-7

$d_1 = 30\left(10^{-3}\right)$ in. $\quad d_2 = 35\left(10^{-3}\right)$ in. $\quad A_1 = \dfrac{\pi}{4}d_1^2 = 7.069 \times 10^{-4}$ in.^{2}

$W = 28$ lb $\quad A_2 = \dfrac{\pi}{4}d_2^2 = 9.621 \times 10^{-4}$ in.^{2}

$\alpha = 22°$ $\quad \beta = 40°$

(a) Find normal stress in wires

$$T_2 = \frac{W}{\dfrac{\cos(\beta)}{\cos(\alpha)}\sin(\alpha) + \sin(\beta)} = 29.403 \text{ lb} \qquad \boxed{\sigma_2 = \frac{T_2}{A_2} = 30.6 \text{ ksi}}$$

$$T_1 = T_2\frac{\cos(\beta)}{\cos(\alpha)} = 24.293 \text{ lb} \qquad \boxed{\sigma_1 = \frac{T_1}{A_1} = 34.4 \text{ ksi}}$$

(b) Find new d_1 s.t. normal stresses in wires is the same

$$A_{1\,\text{new}} = \frac{T_1}{\sigma_2} = 7.949 \times 10^{-4} \text{ in.}^2 \qquad \boxed{d_{1\,\text{new}} = \sqrt{\frac{4}{\pi}A_{1\,\text{new}}} = 3.18 \times 10^{-2} \text{ in.}} \quad \text{or} \quad 31.8 \text{ mils}$$

$$\sigma_{1\text{new}} = \frac{T_1}{\dfrac{\pi}{4}{d_{1\text{new}}}^2} = 30.6 \text{ ksi}$$

(c) Now, to stabilize the camera for windy outdoor conditions, a third wire is added (see figure b); assume the 3 wires meet at a common point (coordinates = (0, 0, 0) above the camera at the instant shown in figure b); wire 1 is attached to a support at coordinates (75', 48', 70'); wire 2 is supported at (−70', 55', 80'); and wire 3 is supported at (−10', −85', 75'); assume that all three wires have diameter of 30 mils. Find tensile stresses in wires 1 to 3.

$d = 30\left(10^{-3}\right)$ in. $\quad A\,\dfrac{\pi}{4}d^2 = 7.069 \times 10^{-4}$ in.^{2}

Position vectors from camera to each support $\quad r_1 = \begin{pmatrix} 75 \\ 48 \\ 70 \end{pmatrix}$ ft $\quad r_2 = \begin{pmatrix} -70 \\ 55 \\ 80 \end{pmatrix}$ ft $\quad r_3 = \begin{pmatrix} -10 \\ -85 \\ 75 \end{pmatrix}$ ft $\quad W = 28\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ lb

$L_1 = |r_1| = 113.265 \qquad L_2 = |r_2| = 119.687 \qquad L_3 = |r_3| = 113.798$

Unit vectors along wires 1 to 3 $\quad e_1 = \dfrac{r_1}{|r_1|} = \begin{pmatrix} 0.662 \\ 0.424 \\ 0.618 \end{pmatrix} \quad e_2 = \dfrac{r_2}{|r_2|} = \begin{pmatrix} -0.585 \\ 0.46 \\ 0.668 \end{pmatrix} \quad e_3 = \dfrac{r_3}{|r_3|} = \begin{pmatrix} -0.088 \\ -0.747 \\ 0.659 \end{pmatrix}$

$T_1 = F_1 e_1 \qquad T_2 = F_2 e_2 \qquad T_3 = F_3 e_3 \qquad i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Equilibrium of forces $\qquad T_1 + T_2 + T_3 = W$

$$T^{\langle 1\rangle} = e_1 \qquad T^{\langle 2\rangle} = e_2 \qquad T^{\langle 3\rangle} = e_3 \qquad T = \begin{pmatrix} 0.662 & -0.585 & -0.088 \\ 0.424 & 0.46 & -0.747 \\ 0.618 & 0.668 & 0.659 \end{pmatrix}$$

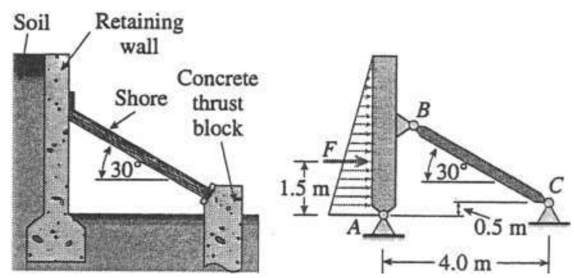
$$F = T^{-1}\,W = \begin{pmatrix} 13.854 \\ 13.277 \\ 16.028 \end{pmatrix} \text{lb} \qquad \sigma_1 = \frac{F_1}{A} = 19.6 \text{ ksi} \qquad \sigma_2 = \frac{F_2}{A} = 18.78 \text{ ksi} \qquad \sigma_3 = \frac{F_3}{A} = 22.7 \text{ ksi}$$

$$\boxed{\sigma_1 = 19.6 \text{ ksi}} \qquad \boxed{\sigma_2 = 18.78 \text{ ksi}} \qquad \boxed{\sigma_3 = 22.7 \text{ ksi}}$$

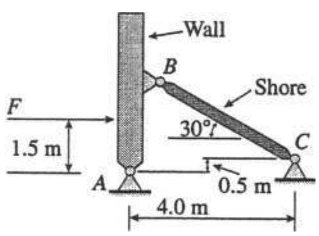
Problem 1.3-8 A long retaining wall is braced by wood shores set at an angle of 30° and supported by concrete thrust blocks, as shown in the first part of the figure. The shores are evenly spaced, 3 m apart.

For analysis purposes, the wall and shores are idealized as shown in the second part of the figure. Note that the base of the wall and both ends of the shores are assumed to be pinned. The pressure of the soil against the wall is assumed to be triangularly distributed, and the resultant force acting on a 3-meter length of the wall is $F = 190$ kN.

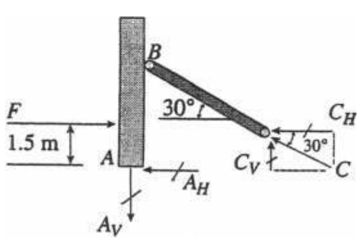
If each shore has a 150 mm × 150 mm square cross section, what is the compressive stress σ_c in the shores?



Solution 1.3-8 Retaining wall braced by wood shores



Free-body diagram of wall and shore



C = compressive force in wood shore

C_H = horizontal component of C

C_V = vertical component of C

$C_H = C \cos 30°$

$C_V = C \sin 30°$

$F = 190$ kN

A = area of one shore

$A = (150 \text{ mm})(150 \text{ mm})$

$= 22{,}500 \text{ mm}^2$

$= 0.0225 \text{ m}^2$

Summation of moments about point A

$\Sigma M_A = 0$

$-F(1.5 \text{ m}) + C_V(4.0 \text{ m}) + C_H(0.5 \text{ m}) = 0$

or

$-(190 \text{ kN})(1.5 \text{ m}) + C(\sin 30°)(4.0 \text{ m}) + C(\cos 30°)(0.5 \text{ m}) = 0$

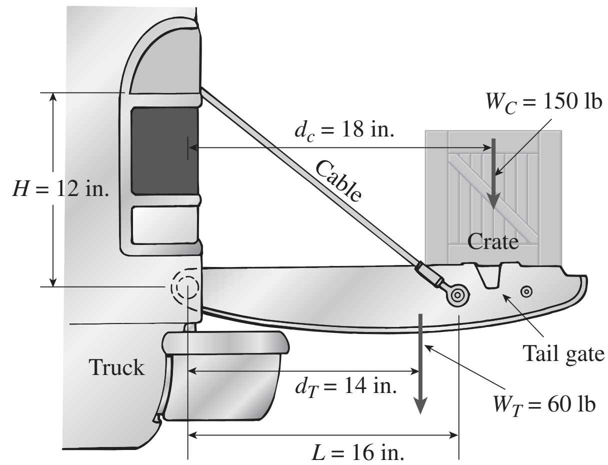
$\therefore C = 117.14$ kN

Compressive stress in the shores

$$\sigma_c = \frac{C}{A} = \frac{117.14 \text{ kN}}{0.0225 \text{ m}^2} = 5.21 \text{ MPa} \quad \leftarrow$$

Problem 1.3-9 A pickup truck tailgate supports a crate ($W_C = 150$ lb), as shown in the figure. The tailgate weighs $W_T = 60$ lb and is supported by two cables (only one is shown in the figure). Each cable has an effective cross-sectional area $A_e = 0.017$ in^2.

(a) Find the tensile force T and normal stress σ in each cable.
(b) If each cable elongates $\delta = 0.01$ in. due to the weight of both the crate and the tailgate, what is the average strain in the cable?



Solution 1.3-9

$W_c = 150$ lb

$A_e = 0.017$ in.^{2}

$W_T = 60$

$\delta = 0.01$

$d_c = 18$

$d_T = 14$

$H = 12$

$L = 16$

$L_c = \sqrt{L^2 + H^2}$ $\qquad L_c = 20$

$\sum M_{\text{hinge}} = 0 \qquad 2T_vL = W_cd_c + W_Td_T$

$T_v = \dfrac{W_cd_c + W_Td_T}{2L} \qquad T_v = 110.625$ lb

$T_h = \dfrac{L}{H}T_v \qquad T_h = 147.5$

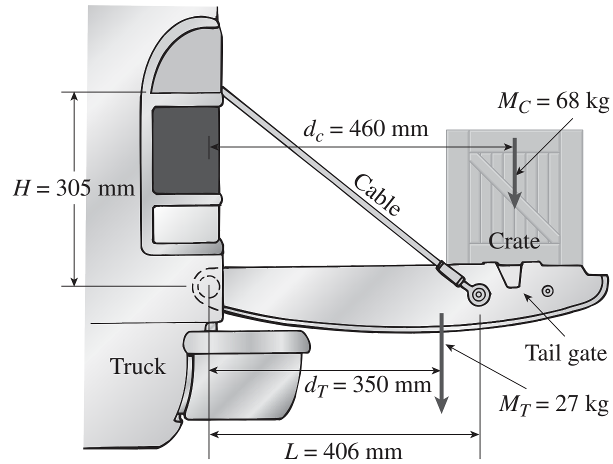
(a) $T = \sqrt{T_v^{\,2} + T_h^{\,2}} \quad T = 184.4$ lb $\leftarrow$

$\sigma_{\text{cable}} = \dfrac{T}{A_e} \qquad \sigma_{\text{cable}} = 10.8$ ksi $\leftarrow$

(b) $\varepsilon_{\text{cable}} = \dfrac{\delta}{L_c} \qquad \varepsilon_{\text{cable}} = 5 \times 10^{-4}$ $\leftarrow$

Problem 1.3-10 Solve the preceding problem if the mass of the tail gate is $M_T = 27$ kg and that of the crate is $M_C = 68$ kg. Use dimensions $H = 305$ mm, $L = 406$ mm, $d_C = 460$ mm, and $d_T = 350$ mm. The cable cross-sectional area is $A_e = 11.0 \text{ mm}^2$.

(a) Find the tensile force T and normal stress σ in each cable.
(b) If each cable elongates $\delta = 0.25$ mm due to the weight of both the crate and the tailgate, what is the average strain in the cable?



Solution 1.3-10

$M_c = 68$

$M_T = 27$ kg $\qquad g = 9.81 \text{ m/s}^2$

$W_c = M_c g \qquad W_T = M_T g$

$W_c = 667.08 \qquad W_T = 264.87$

$N = \text{kg}\cdot\text{m/s}^2$

$A_e = 11.0 \text{ mm}^2 \qquad \delta = 0.25$

$d_c = 460 \qquad d_T = 350$

$H = 305 \qquad L = 406$

$L_c = \sqrt{L^2 + H^2} \qquad L_c = 507.8$ mm

$\sum M_{\text{hinge}} = 0 \qquad 2T_v L = W_c d_c + W_T d_T$

$$T_v = \frac{W_c d_c + W_T d_T}{2L} \qquad T_v = 492.071 \text{ N}$$

$$T_h = \frac{L}{H} T_v \qquad T_h = 655.019 \text{ N}$$

(a) $T = \sqrt{T_v^2 + T_h^2} \qquad T = 819 \text{ N} \quad \leftarrow$

$$\sigma_{\text{cable}} = \frac{T}{A_e} \qquad \sigma_{\text{cable}} = 74.5 \text{ MPa} \quad \leftarrow$$

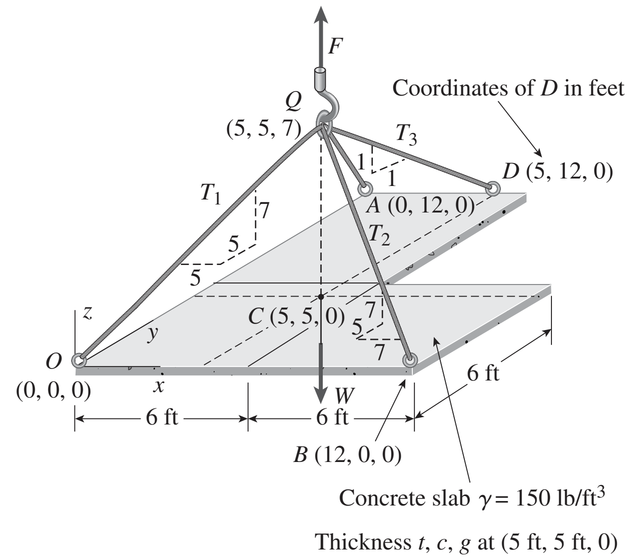
(b) $\varepsilon_{\text{cable}} = \dfrac{\delta}{L_c} \qquad \varepsilon_{\text{cable}} = 4.92 \times 10^{-4} \quad \leftarrow$

Problem 1.3-11 An L-shaped reinforced concrete slab 12 ft × 12 ft (but with a 6 ft × 6 ft cutout) and thickness $t = 9.0$ in. is lifted by three cables attached at O, B and D, as shown in the figure. The cables are combined at point Q, which is 7.0 ft above the top of the slab and directly above the center of mass at C. Each cable has an effective cross-sectional area of $A_e = 0.12$ in.^{2}.

(a) Find the tensile force $T_1 (i = 1, 2, 3)$ in each cable due to the weight W of the concrete slab (ignore weight of cables).

(b) Find the average stress σ_i in each cable. (see Table I-1 in Appendix I for the weight density of reinforced concrete.)

(c) Add cable AQ so that OQA is one continuous cable, with each segment having force T_1, which is connected to cables BQ and DQ at point Q. Repeat parts (a) and (b). (*Hint*: There are now three force equilibrium equations and one *constraint equation*, $T_1 = T_4$)



Solution 1.3-11

Cable lengths (ft)

$L_1 = \sqrt{5^2 + 5^2 + 7^2}$ $L_1 = 9.95$ $L_2 = \sqrt{5^2 + 7^2 + 7^2}$ $L_2 = 11.091$ $L_3 = \sqrt{7^2 + 7^2}$ $L_3 = 9.899$

(a) Solution for cable forces using statics (three equations, three unknowns); units = lb, ft

$$r_{OQ} = \begin{pmatrix} 5 \\ 5 \\ 7 \end{pmatrix} \qquad r_{BQ} = \begin{pmatrix} -7 \\ 5 \\ 7 \end{pmatrix} \qquad r_{DQ} = \begin{pmatrix} 0 \\ -7 \\ 7 \end{pmatrix}$$

$$e_{OQ} = \frac{r_{OQ}}{|r_{OQ}|} = \begin{pmatrix} 0.503 \\ 0.503 \\ 0.704 \end{pmatrix} \qquad e_{BQ} = \frac{r_{BQ}}{|r_{BQ}|} = \begin{pmatrix} -0.631 \\ 0.451 \\ 0.631 \end{pmatrix} \qquad e_{DQ} = \frac{r_{DQ}}{|r_{DQ}|} = \begin{pmatrix} 0 \\ -0.707 \\ 0.707 \end{pmatrix}$$

$$W = 150\,(12^2 - 6^2)\frac{9}{12} = 12{,}150 \text{ lbs}$$

Statics $\quad \Sigma F = 0 \quad T_1 e_{OQ} + T_2 e_{BQ} + T_3 e_{DQ} - \begin{pmatrix} 0 \\ 0 \\ W \end{pmatrix} = \begin{pmatrix} 0.50252\,T_1 - 0.63117\,T_2 \\ 0.50252\,T_1 + 0.45083\,T_2 - 0.70711\,T_3 \\ 0.70353\,T_1 + 0.63117\,T_2 + 0.70711\,T_3 - 12{,}150 \end{pmatrix}$

or in matrix form; solve simultaneous equations to get cable tension forces

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} e_{OQ_{1,1}} & e_{BQ_{1,1}} & e_{DQ_{1,1}} \\ e_{OQ_{2,1}} & e_{BQ_{2,1}} & e_{DQ_{2,1}} \\ e_{OQ_{3,1}} & e_{BQ_{3,1}} & e_{DQ_{3,1}} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ W \end{pmatrix} = \begin{pmatrix} 5877 \\ 4679 \\ 7159 \end{pmatrix} \text{lb} \qquad \boxed{T = \begin{pmatrix} 5877 \\ 4679 \\ 7159 \end{pmatrix} \text{lb}}$$

(b) Average normal stress in each cable

$$i = 1 \ldots 3 \qquad \sigma_i = \frac{T_i}{A_e} \qquad \boxed{\sigma = \begin{pmatrix} 48975 \\ 38992 \\ 59658 \end{pmatrix} \text{psi}} \qquad A_e = 0.12 \text{ in.}^2$$

(c) ADD CONTINUOUS CABLE OQA

$$r_{OQ} = \begin{pmatrix} 5 \\ 5 \\ 7 \end{pmatrix} \quad r_{AQ} = \begin{pmatrix} 5 \\ -7 \\ 7 \end{pmatrix} \quad r_{BQ} = \begin{pmatrix} -7 \\ 5 \\ 7 \end{pmatrix} \quad r_{DQ} = \begin{pmatrix} 0 \\ -7 \\ 7 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.451 \\ -0.631 \\ 0.631 \end{pmatrix}$$

$$e_{OQ} = \frac{r_{OQ}}{|r_{OQ}|} = \begin{pmatrix} 0.503 \\ 0.503 \\ 0.704 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.451 \\ -0631 \\ 0.631 \end{pmatrix} \quad e_{BQ} = \frac{r_{BQ}}{|r_{BQ}|} = \begin{pmatrix} -0.631 \\ 0.451 \\ 0.631 \end{pmatrix}$$

STATICS Solve simultaneous equations to get cable tension forces

$$\begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{pmatrix} = \begin{pmatrix} e_{OQ_{1,1}} & e_{BQ_{1,1}} & e_{DQ_{1,1}} & e_{AQ_{1,1}} \\ e_{OQ_{2,1}} & e_{BQ_{2,1}} & e_{DQ_{2,1}} & e_{AQ_{2,1}} \\ e_{OQ_{3,1}} & e_{BQ_{3,1}} & e_{DQ_{3,1}} & e_{AQ_{3,1}} \\ 1 & 0 & 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ W \\ 0 \end{pmatrix} = \begin{pmatrix} 4278 \\ 6461 \\ 3341 \\ 4278 \end{pmatrix} \text{lbs} \quad \boxed{T = \begin{pmatrix} 4278 \\ 6461 \\ 3341 \\ 4278 \end{pmatrix} \text{lb}}$$

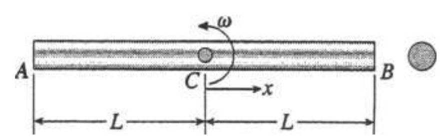
< for case of $T_1 = T_4$

Normal stresses in cables

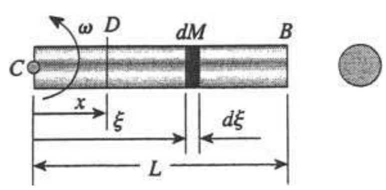
$$i = 1 \ldots 4 \qquad \sigma_i = \frac{T_i}{A_e} \qquad \boxed{\sigma = \begin{pmatrix} 35650 \\ 53842 \\ 27842 \\ 35650 \end{pmatrix} \text{psi}}$$

Problem 1.3-12 A round bar ACB of length $2L$ (see figure) rotates about an axis through the midpoint C with constant angular speed ω (radians per second). The material of the bar has weight density γ.

(a) Derive a formula for the tensile stress σ_x in the bar as a function of the distance x from the midpoint C.
(b) What is the maximum tensile stress σ_{max}?



Solution 1.3-12 Rotating Bar



ω = angular speed (rad/s)

A = cross-sectional area

γ = weight density

$\dfrac{\gamma}{g}$ = mass density

We wish to find the axial force F_x in the bar at Section D, distance x from the midpoint C.

The force F_x equals the inertia force of the part of the rotating bar from D to B.

Consider an element of mass dM at distance ξ from the midpoint C. The variable ξ ranges from x to L.

$$dM = \frac{\gamma}{g} A \, dj$$

dF = Inertia force (centrifugal force) of element of mass dM

$$dF = (dM)(j\omega^2) = \frac{\gamma}{g} A\omega^2 j dj$$

$$F_x = \int_D^B dF = \int_x^L \frac{\gamma}{g} A\omega^2 j dj = \frac{\gamma A \omega^2}{2g}(L^2 - x^2)$$

(a) TENSILE STRESS IN BAR AT DISTANCE x

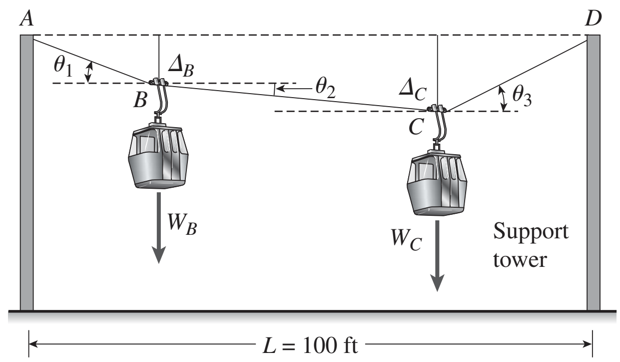
$$\sigma_x = \frac{F_x}{A} = \frac{\gamma\omega^2}{2g}(L^2 - x^2) \quad \leftarrow$$

(b) MAXIMUM TENSILE STRESS

$$x = 0 \quad \sigma_{max} = \frac{\gamma\omega^2 L^2}{2g} \quad \leftarrow$$

Problem 1.3-13 Two gondolas on a ski lift are locked in the position shown in the figure while repairs are being made elsewhere. The distance between support towers is $L = 100$ ft. The length of each cable segment under gondola weights $W_B = 450$ lb and $W_C = 650$ lb are $D_{AB} = 12$ ft, $D_{BC} = 70$ ft, and $D_{CD} = 20$ ft. The cable sag at B is $\Delta_B = 3.9$ ft and that at $C(\Delta_C)$ is 7.1 ft. The effective cross-sectional area of the cable is $A_e = 0.12$ in^2.

(a) Find the tension force in each cable segment; neglect the mass of the cable.
(b) Find the average stress (σ) in each cable segment.



Solution 1.3-13

$W_B = 450$

$Wc = 650$ lb

$\Delta_B = 3.9$ ft

$\Delta_C = 7.1$ ft

$L = 100$ ft

$D_{AB} = 12$ ft

$D_{BC} = 70$ ft

$D_{CD} = 20$ ft

$D_{AB} + D_{BC} + D_{CD} = 102$ ft

$A_e = 0.12$ in.^{2}

Compute initial values of theta angles (radians)

$$\theta_1 = \arcsin\left(\frac{\Delta_B}{D_{AB}}\right) \qquad \theta_1 = 0.331$$

$$\theta_2 = \arcsin\left(\frac{\Delta_C - \Delta_B}{D_{BC}}\right) \qquad \theta_2 = 0.046$$

$$\theta_3 = \arcsin\left(\frac{\Delta_C}{D_{CD}}\right) \qquad \theta_3 = 0.363$$

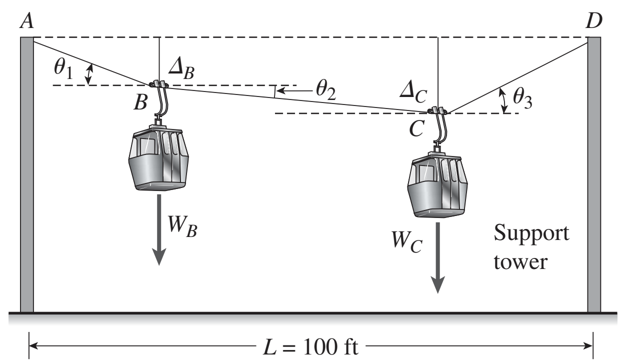
(a) Statics at B and C

$$-T_{AB}\cos(\theta_1) + T_{BC}\cos(\theta_2) = 0$$

$$T_{AB}\sin(\theta_1) - T_{BC}\sin(\theta_2) = W_B$$

$$-T_{BC}\cos(\theta_2) + T_{CD}\cos(\theta_3) = 0$$

$$T_{BC}\sin(\theta_2) + T_{CD}\sin(\theta_3) = W_C$$



Constraint equations

$$D_{AB}\cos(\theta_1) + D_{BC}\cos(\theta_2) + D_{CD}\cos(\theta_3) = L$$

$$D_{AB}\sin(\theta_1) + D_{BC}\sin(\theta_2) = D_{CD}\sin(\theta_3)$$

Solve simultaneous equations numerically for tension force in each cable segment

$T_{AB} = 1620$ lb $\quad T_{CB} = 1536$ lb $\quad T_{CD} = 1640$ lb $\quad \leftarrow$

Check equilibrium at B and C

$$T_{AB}\sin(\theta_1) - T_{BC}\sin(\theta_2) = 450$$

$$T_{BC}\sin(\theta_2) + T_{CD}\sin(\theta_3) = 650$$

(b) Compute stresses in cable segments

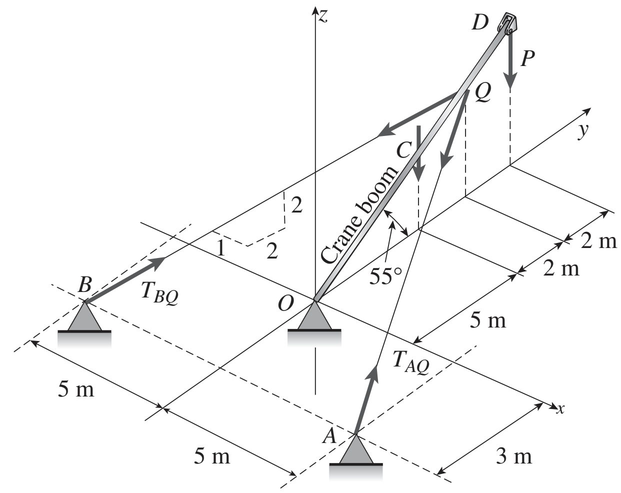
$$\sigma_{AB} = \frac{T_{AB}}{A_e} \qquad \sigma_{BC} = \frac{T_{BC}}{A_e} \qquad \sigma_{CD} = \frac{T_{CD}}{A_e}$$

$\sigma_{AB} = 13.5$ ksi $\quad \sigma_{BC} = 12.8$ ksi

$\sigma_{CD} = 13.67$ ksi $\quad \leftarrow$

Problem 1.3-14 A crane boom of mass 450 kg with its center of mass at C is stabilized by two cables AQ and BQ ($A_e = 304\ \text{mm}^2$ for each cable) as shown in the figure. A load $P = 20$ kN is supported at point D. The crane boom lies in the y–z plane.

(a) Find the tension forces in each cable: T_{AQ} and T_{BQ} (kN). Neglect the mass of the cables, but include the mass of the boom in addition to load P.
(b) Find the average stress (σ) in each cable.



Solution 1.3-14

Data $\quad M_{\text{boom}} = 450$ kg

$g = 9.81\ \text{m/s}^2 \qquad W_{\text{boom}} = M_{\text{boom}}\, g$

$W_{\text{boom}} = 4415$ N

$P = 20$ kN

$A_e = 304\ \text{mm}^2$

(a) Symmetry: $T_{AQ} = T_{BQ}$

$$\sum M_x = 0$$

$$2T_{AQZ}(3000) = W_{\text{boom}}(5000) + P(9000)$$

$$T_{AQZ} = \frac{W_{\text{boom}}(5000) + P(9000)}{2(3000)}$$

$$T_{AQ} = \frac{\sqrt{2^2 + 2^2 + 1^2}}{2} T_{AQz}$$

$$T_{AQ} = 50.5\ \text{kN} = T_{BQ} \quad \leftarrow$$

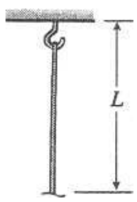
(b) $\sigma = \dfrac{T_{AQ}}{A_e} \qquad \sigma = 166.2\ \text{MPa} \quad \leftarrow$

Mechanical Properties of Materials

Problem 1.4-1 Imagine that a long steel wire hangs vertically from a high-altitude balloon.

(a) What is the greatest length (feet) it can have without yielding if the steel yields at 40 ksi?
(b) If the same wire hangs from a ship at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Appendix I, Table I-1.)

Solution 1.4-1 Hanging wire of length *L*



W = total weight of steel wire

γ_S = weight density of steel
$= 490 \text{ lb/ft}^3$

γ_w = weight density of sea water
$= 63.8 \text{ lb/ft}^3$

A = cross-sectional area of wire

$\sigma_{max} = 40$ ksi (yield strength)

(a) Wire hanging in air

$$W = \gamma_S AL$$

$$\sigma_{max} = \frac{W}{A} = \gamma_S L$$

$$L_{max} = \frac{\sigma_{max}}{\gamma_S} = \frac{40{,}000 \text{ psi}}{490 \text{ lb/ft}^3}\left(144 \text{ in.}^2/\text{ft}^2\right)$$

$$= 11{,}800 \text{ ft} \quad \leftarrow$$

(b) Wire hanging in sea water

F = tensile force at top of wire

$$F = \left(\gamma_S - \gamma_W\right)AL \quad \sigma_{max} = \frac{F}{A} = \left(\gamma_S - \gamma_W\right)L$$

$$L_{max} = \frac{\sigma_{max}}{\gamma_S - \gamma_W}$$

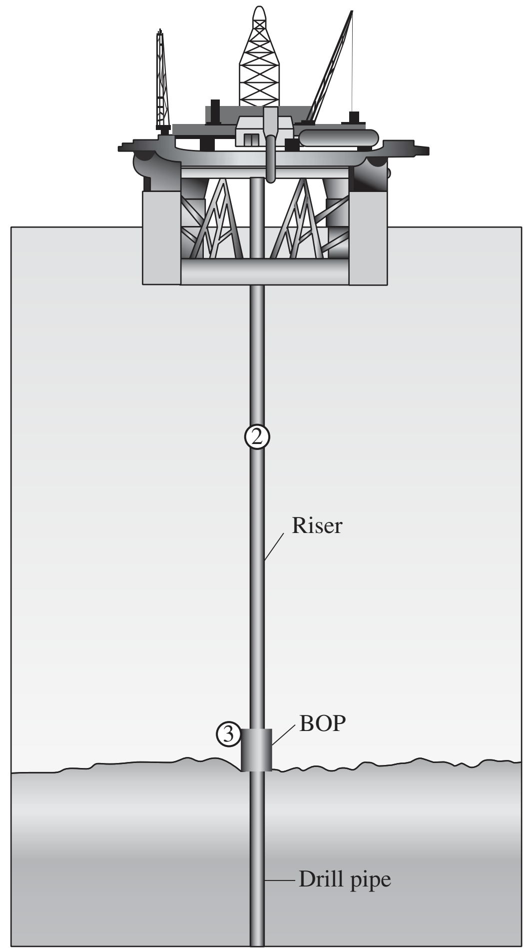
$$= \frac{40{,}000 \text{ psi}}{(490 - 63.8) \text{ lb/ft}^3}\left(144 \text{ in.}^2/\text{ft}^2\right)$$

$$= 13{,}500 \text{ ft} \quad \leftarrow$$

Problem 1.4-2 Steel riser pipe hangs from a drill rig located offshore in deep water (see figure).

(a) What is the greatest length (meters) it can have without breaking if the pipe is suspended in air and the ultimate strength (or breaking strength) is 550 MPa?

(b) If the same riser pipe hangs from a drill rig at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Appendix I, Table I-1. Neglect the effect of buoyant foam casings on the pipe).



Solution 1.4-2

(a) PIPE SUSPENDED IN AIR

$$\sigma_U = 550 \text{ MPa}$$

$$\gamma_s = 77 \text{ kN/m}^3$$

$$W = \gamma_s AL$$

$$\boxed{L_{\max} = \frac{\sigma_U}{\gamma_s} = 7143\,\text{m}}$$

(b) PIPE SUSPENDED IN SEA WATER

$$\gamma_w = 10\,\text{kN/m}^3$$

Force at top of pipe: $F = (\gamma_s - \gamma_w)AL$

Stress at top of pipe:

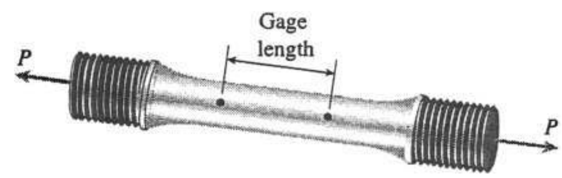
$$\sigma_{\max} = \frac{F}{A} \qquad \sigma_{\max} = (\gamma_s - \gamma_w)L$$

Set max stress equal to ultimate and then solve for $L_{\max}$

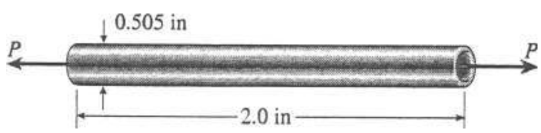
$$\boxed{L_{\max} = \frac{\sigma_U}{(\gamma_s - \gamma_w)} = 8209\,\text{m}}$$

Problem 1.4-3 Three different materials, designated *A, B*, and *C*, are tested in tension using test specimens having diameters of 0.505 in. and gage lengths of 2.0 in. (see figure). At failure, the distances between the gage marks are found to be 2.13, 2.48, and 2.78 in., respectively. Also, at the failure cross sections the diameters are found to be 0.484, 0.398, and 0.253 in., respectively.

Determine the percent elongation and percent reduction in area of each specimen, and then, using your own judgment, classify each material as brittle or ductile.



Solution 1.4-3 Tensile tests of three materials



$$\text{Percent elongation} = \frac{L_1 - L_0}{L_0}(100) = \left(\frac{L_1}{L_0} - 1\right)100$$

$L_0 = 2.0$ in.

$$\text{Percent elongation} = \left(\frac{L_1}{2.0} - 1\right)(100) \qquad \text{(Eq. 1)}$$

where L_1 is in inches.

$$\text{Percent reduction in area} = \frac{A_0 - A_1}{A_0}(100) = \left(1 - \frac{A_1}{A_0}\right)(100)$$

d_0 = initial diameter $\qquad d_1$ = final diameter

$$\frac{A_1}{A_0} = \left(\frac{d_1}{d_0}\right)^2 \qquad d_0 = 0.505 \text{ in.}$$

Percent reduction in area

$$= \left[1 - \left(\frac{d_1}{0.505}\right)^2\right](100) \qquad \text{(Eq. 2)}$$

where d_1 is in inches.

Material	L_1 (in.)	d_1 (in.)	% Elongation (Eq. 1)	% Reduction (Eq. 2)	Brittle or Ductile?
A	2.13	0.484	6.5%	8.1%	Brittle
B	2.48	0.398	24.0%	37.9%	Ductile
C	2.78	0.253	39.0%	74.9%	Ductile

Problem 1.4-4 The *strength-to-weight ratio* of a structural material is defined as its load-carrying capacity divided by its weight. For materials in tension, we may use a characteristic tensile stress (as obtained from a stress-strain curve) as a measure of strength. For instance, either the yield stress or the ultimate stress could be used, depending upon the particular application. Thus, the strength-to-weight ratio $R_{S/W}$ for a material in tension is defined as

$$R_{S/W} = \frac{\sigma}{\gamma}$$

in which σ is the characteristic stress and γ is the weight density. Note that the ratio has units of length.

Using the ultimate stress σ_U as the strength parameter, calculate the strength-to-weight ratio (in units of meters) for each of the following materials: aluminum alloy 6061-T6, Douglas fir (in bending), nylon, structural steel ASTM-A572, and a titanium alloy. (Obtain the material properties from Appendix I, Tables I-1 and I-3. When a range of values is given in a table, use the average value.)

Solution 1.4-4 Strength-to-weight ratio

The ultimate stress σ_U for each material is obtained from Appendix I, Tables I-3, and the weight density γ is obtained from Table I-1.

The strength-to-weight ratio (meters) is

$$R_{S/W} = \frac{\sigma_U\,(\text{MPa})}{\gamma(\text{kN/m}^3)}\,(10^3)$$

Values of σ_U, γ, and $R_{S/W}$ are listed in the table.

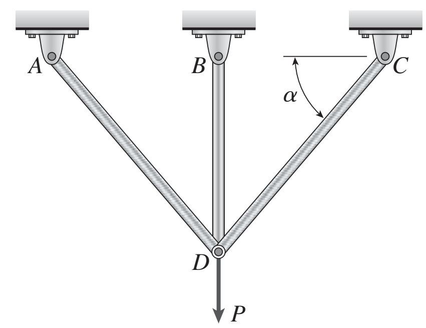
	σ_U (MPa)	γ (kN/m^3)	$R_{S/W}$ (m)
Aluminum alloy 6061-T6	310	26.0	11.9×10^3
Douglas fir	65	5.1	12.7×10^3
Nylon	60	9.8	6.1×10^3
Structural steel ASTM-A572	500	77.0	6.5×10^3
Titanium alloy	1050	44.0	23.9×10^3

Titanium has a high strength-to-weight ratio, which is why it is used in space vehicles and high-performance airplanes. Aluminum is higher than steel, which makes it desirable for commercial aircraft. Some woods are also higher than steel, and nylon is about the same as steel.

Problem 1.4-5 A symmetrical framework consisting of three pin-connected bars is loaded by a force P (see figure). The angle between the inclined bars and the horizontal is $\alpha = 52°$. The axial strain in the middle bar is measured as 0.036.

Determine the tensile stress in the outer bars if they are constructed of a copper alloy having the following stress-strain relationship:

$$\sigma = \frac{18{,}000\varepsilon}{1 + 300\varepsilon} \qquad 0 \le \varepsilon \le 0.03\ (\sigma = \text{ksi})$$



Solution 1.4-5

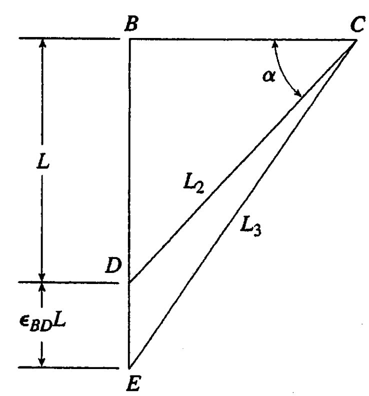
Data

$\varepsilon_{BD} = 0.036 \qquad \alpha = 52° \qquad L_{BD} = 1$ < assume unit length to facilitate numerical calculations below

Strain in CE

$$\varepsilon_{CE} = \frac{L_3 - L_2}{L_2}$$

$$L_2 = \frac{L_{BD}}{\sin(\alpha)} \qquad L_{BC} = \frac{L_{BD}}{\tan(\alpha)}$$



Increased length of *CE* (see figure)

$$L_3 = \sqrt{L_{BC}{}^2 + (L_{BD} + \varepsilon_{BD} L_{BD})^2} = \sqrt{\frac{1}{\tan(52°)^2} + 1.073296}$$

Compute strain in *CE* then substitute strain value into stress-strain relationship to find tensile stress in outer bars:

$$\varepsilon_{CE} = \frac{L_3 - L_2}{L_2} = 0.023 \qquad \sigma = \frac{18000\,\varepsilon_{CE}}{1 + 300\,\varepsilon_{CE}} \qquad \boxed{\sigma = 52.3 \quad \text{ksi}}$$

Problem 1.4-6 A specimen of a methacrylate plastic is tested in tension at room temperature (see figure), producing the stress-strain data listed in the accompanying table.

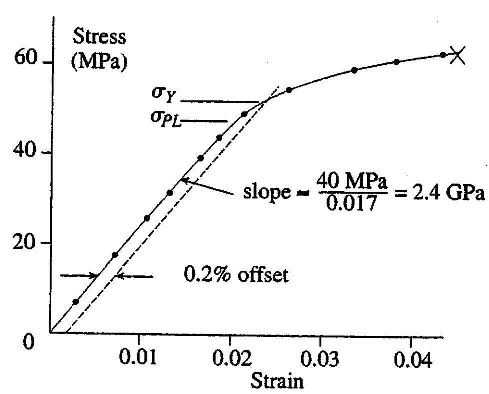
Plot the stress-strain curve and determine the proportional limit, modulus of elasticity (i.e., the slope of the initial part of the stress-strain curve), and yield stress at 0.2% offset. Is the material ductile or brittle?

STRESS-STRAIN DATA FOR PROBLEM 1.4-6

Stress (MPa)	Strain
8.0	0.0032
17.5	0.0073
25.6	0.0111
31.1	0.0129
39.8	0.0163
44.0	0.0184
48.2	0.0209
53.9	0.0260
58.1	0.0331
62.0	0.0429
62.1	Fracture

Solution 1.4-6 Tensile test of a plastic

Using the stress-strain data given in the problem statement, plot the stress-strain curve:



σ_{PL} = proportional limit $\quad \sigma_{PL} \approx 47$ MPa ←

Modulus of elasticity (slope) ≈ 2.4 GPa ←

σ_Y = yield stress at 0.2% offset

$\sigma_Y \approx 53$ MPa ←

Material is *brittle*, because the strain after the proportional limit is exceeded is relatively small. ←

Problem 1.4-7 The data shown in the accompanying table were obtained from a tensile test of high-strength steel. The test specimen had a diameter of 0.505 in. and a gage length of 2.00 in. (see figure for Prob. 1.4-3). At fracture, the elongation between the gage marks was 0.12 in. and the minimum diameter was 0.42 in.

Plot the conventional stress-strain curve for the steel and determine the proportional limit, modulus of elasticity (i.e., the slope of the initial part of the stress-strain curve), yield stress at 0.1% offset, ultimate stress, percent elongation in 2.00 in., and percent reduction in area.

TENSILE-TEST DATA FOR PROBLEM 1.4-7

Load (lb)	Elongation (in.)
1,000	0.0002
2,000	0.0006
6,000	0.0019
10,000	0.0033
12,000	0.0039
12,900	0.0043
13,400	0.0047
13,600	0.0054
13,800	0.0063
14,000	0.0090
14,400	0.0102
15,200	0.0130
16,800	0.0230
18,400	0.0336
20,000	0.0507
22,400	0.1108
22,600	Fracture

Solution 1.4-7 Tensile test of high-strength steel

$d_0 = 0.505$ in. $\quad L_0 = 2.00$ in.

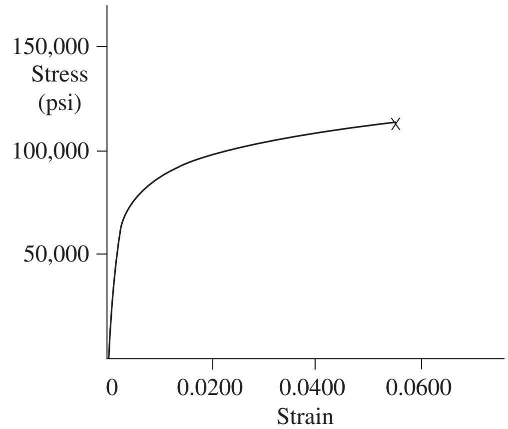
$$A_0 = \frac{\pi d_0^2}{4} = 0.200 \text{ in.}^2$$

CONVENTIONAL STRESS AND STRAIN

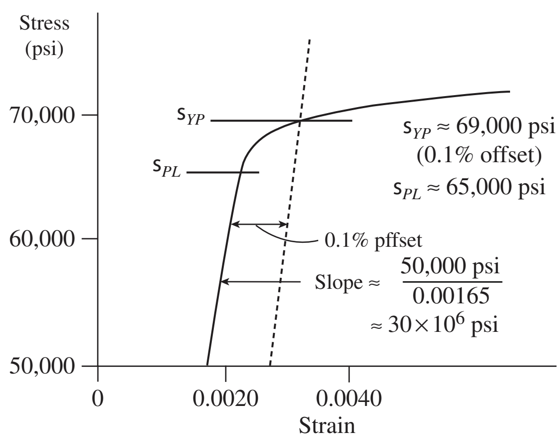
$$\sigma = \frac{P}{A_0} \quad \varepsilon = \frac{\delta}{L_0}$$

Load P (lb)	Elongation δ (in.)	Stress σ (psi)	Strain ε
1,000	0.0002	5,000	0.00010
2,000	0.0006	10,000	0.00030
6,000	0.0019	30,000	0.00100
10,000	0.0033	50,000	0.00165
12,000	0.0039	60,000	0.00195
12,900	0.0043	64,500	0.00215
13,400	0.0047	67,000	0.00235
13,600	0.0054	68,000	0.00270
13,800	0.0063	69,000	0.00315
14,000	0.0090	70,000	0.00450
14,400	0.0102	72,000	0.00510
15,200	0.0130	76,000	0.00650
16,800	0.0230	84,000	0.01150
18,400	0.0336	92,000	0.01680
20,000	0.0507	100,000	0.02535
22,400	0.1108	112,000	0.05540
22,600	Fracture	113,000	

Stress-strain diagram



Enlargement of part of the stress-strain curve



Results

Proportional limit ≈ 65,000 psi ←

Modulus of elasticity (slope) ≈ 30×10^6 psi ←

Yield stress at 0.1% offset ≈ 69,000 psi ←

Ultimate stress (maximum stress)

≈ 113,000 psi ←

Percent elongation in 2.00 in.

$$= \frac{L_1 - L_0}{L_0}(100)$$

$$= \frac{0.12 \text{ in.}}{2.00 \text{ in.}}(100) = 6\% \quad \leftarrow$$

Percent reduction in area

$$= \frac{A_0 - A_1}{A_0}(100)$$

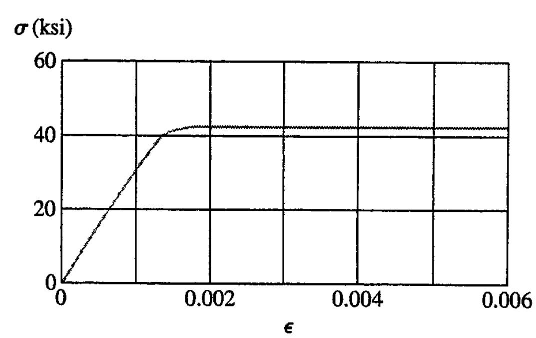
$$= \frac{0.200 \text{ in.}^2 - \frac{\pi}{4}(0.42 \text{ in.})^2}{0.200 \text{ in.}^2}(100)$$

$$= 31\% \quad \leftarrow$$

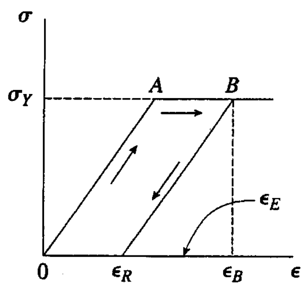
Elasticity, Plasticity, and Creep

Problem 1.5-1 A bar made of structural steel having the stress-strain diagram shown in the figure has a length of 48 in. The yield stress of the steel is 42 ksi and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 30×10^3 ksi. The bar is loaded axially until it elongates 0.20 in., and then the load is removed.

How does the final length of the bar compare with its original length? (*Hint*: Use the concepts illustrated in Fig. 1-36b.)



Solution 1.5-1 Steel bar in tension



$L = 48$ in.

Yield stress $\sigma_Y = 42$ ksi

Slope $= 30 \times 10^3$ ksi

$\delta = 0.20$ in.

STRESS AND STRAIN AT POINT B

$\sigma_B = \sigma_Y = 42$ ksi

$$\varepsilon_B = \frac{\delta}{L} = \frac{0.20 \text{ in.}}{48 \text{ in.}} = 0.00417$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{42 \text{ ksi}}{30 \times 10^3 \text{ ksi}} = 0.00140$$

RESIDUAL STRAIN ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.00417 - 0.00140$$
$$= 0.00277$$

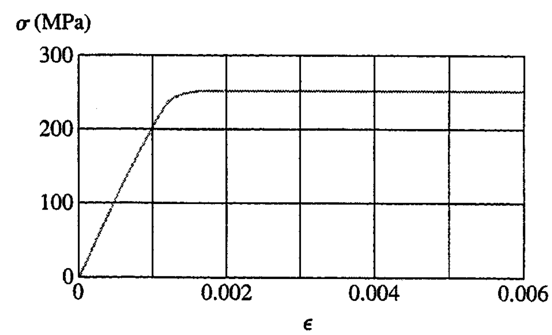
PERMANENT SET

$$\varepsilon_R L = (0.00277)(48 \text{ in.})$$
$$= 0.13 \text{ in.}$$

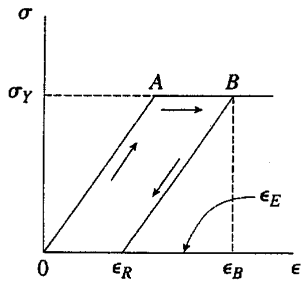
Final length of bar is 0.13 in. greater than its original length. $\longleftarrow$

Problem 1.5-2 A bar of length 2.0 m is made of a structural steel having the stress-strain diagram shown in the figure. The yield stress of the steel is 250 MPa and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 200 GPa. The bar is loaded axially until it elongates 6.5 mm, and then the load is removed.

How does the final length of the bar compare with its original length? (*Hint*: Use the concepts illustrated in Fig. 1-36b.)



Solution 1.5-2 Steel bar in tension



$L = 2.0\text{ m} = 2000\text{ mm}$

Yield stress $\sigma_Y = 250\text{ MPa}$

Slope $= 200\text{ GPa}$

$\delta = 6.5\text{ mm}$

STRESS AND STRAIN AT POINT B

$\sigma_B = \sigma_Y = 250\text{ MPa}$

$$\varepsilon_B = \frac{\delta}{L} = \frac{6.5\text{ mm}}{2000\text{ mm}} = 0.00325$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{250\text{ MPa}}{200\text{ GPa}} = 0.00125$$

RESIDUAL STRAIN ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.00325 - 0.00125$$
$$= 0.00200$$

$$\text{Permanent set} = \varepsilon_R L = (0.00200)(2000\text{ mm})$$
$$= 4.0\text{ mm}$$

Final length of bar is 4.0 mm greater than its original length. ←

Problem 1.5-3 An aluminum bar has length $L = 6$ ft and diameter $d = 1.375$ in. The stress-strain curve for the aluminum is shown in Fig. 1-31 of Section 1.4. The initial straight-line part of the curve has a slope (modulus of elasticity) of 10.6×10^6 psi. The bar is loaded by tensile forces $P = 44.6$ k and then unloaded.

(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit? (*Hint:* Use the concepts illustrated in Figs. 1-36b and 1-37.)

Solution 1.5-3

DATA

$P = 44.6\text{ kip}$ $\quad L = 6\text{ ft}$

$d = 1.375\text{ in.}$ $\quad E = 10.6\,(10^6)\text{ psi}$

NORMAL STRESS IN BAR

$$\sigma_B = \frac{P}{\frac{\pi}{4}d^2} = 30036\text{ psi}$$

from curve, say that $\varepsilon_B = 0.025$

ELASTIC RECOVERY unloading parallel to initial straight line

$$\varepsilon_E = \frac{\sigma_B}{E} = 2.834 \times 10^{-3}$$

RESIDUAL STRAIN

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.022$$

(a) PERMANENT SET

$\varepsilon_R L = 1.596$ in.

(b) PROPORTIONAL LIMIT WHEN RELOADED IS $\sigma_B = 30$ ksi

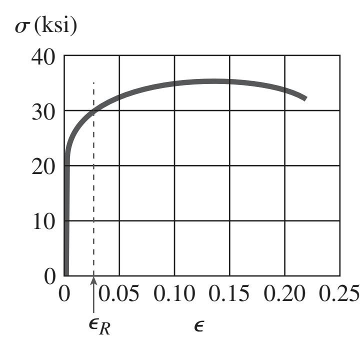


FIG 1-31 Typical stress-strain diagram for an aluminum alloy

Problem 1.5-4 A circular bar of magnesium alloy is 750 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 6.0 mm, and then the load is removed.

(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit? (*Hint:* Use the concepts illustrated in Figs. 1-36b and 1-37.)

Solution 1.5-4

numerical data $L = 750 \text{ mm}$ $\delta = 6 \text{ mm}$

$$E_t(\varepsilon) = \frac{d}{d\varepsilon}\frac{\alpha\,\varepsilon}{1 + \beta\varepsilon} \qquad E_t(0) = 41000 \text{ MPa}$$

(or 41 GPa > magnesium alloy)

$$\varepsilon_B = \frac{\delta}{L} = 8 \times 10^{-3} \qquad \sigma_B 65.6 \text{ MPa} < \text{from curve (see figure)}$$

$\varepsilon_E = 0.0023$ < elastic recovery (see figure)

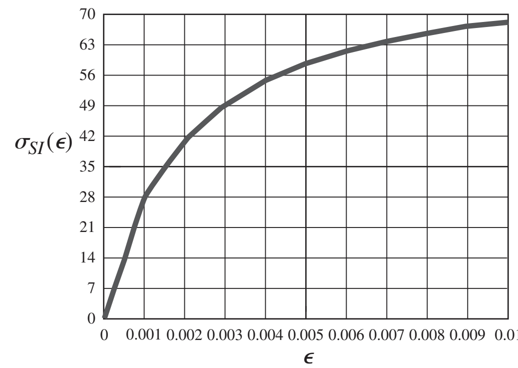
$\varepsilon_R = \varepsilon_B - \varepsilon_E = 5.7 \times 10^{-3}$ < residual strain

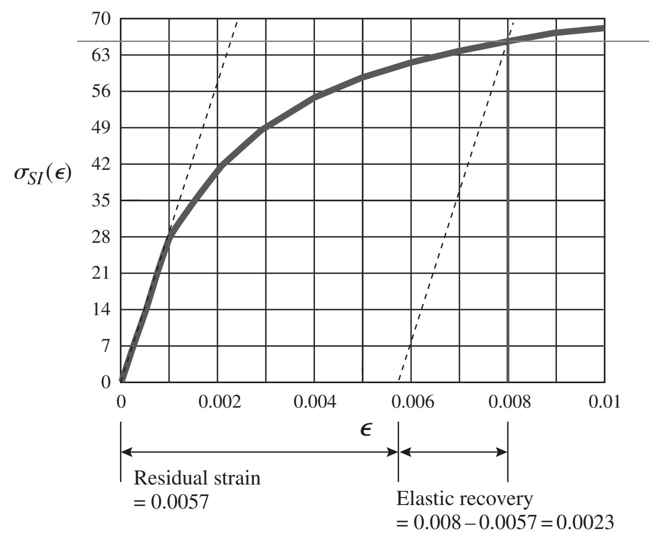
(a) PERMANENT SET

$\delta_{pset} = \varepsilon_R L = 4.275$ $\boxed{\delta_{pset} = 4.28 \text{ mm}}$

(b) PROPORTIONAL LIMIT WHEN RELOADED

$\boxed{\sigma_B = 65.6 \text{ MPa}}$





Problem 1.5-5 A wire of length $L = 4$ ft and diameter $d = 0.125$ in. is stretched by tensile forces $P = 600$ lb. The wire is made of a copper alloy having a stress-strain relationship that may be described mathematically by the following equation:

$$\sigma = \frac{18{,}000\epsilon}{1 + 300\epsilon} \quad 0 \le \epsilon \le 0.03 \quad (\sigma = \text{ksi})$$

in which ϵ is nondimensional and σ has units of kips per square inch (ksi).

(a) Construct a stress-strain diagram for the material.
(b) Determine the elongation of the wire due to the forces P.
(c) If the forces are removed, what is the permanent set of the bar?
(d) If the forces are applied again, what is the proportional limit?

Solution 1.5-5 Wire stretched by forces *P*

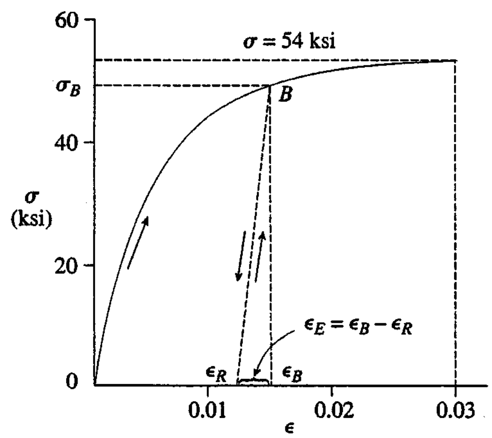
$L = 4\text{ ft} = 48\text{ in.} \qquad d = 0.125\text{ in.}$

$P = 600\text{ lb}$

COPPER ALLOY

$$\sigma = \frac{18{,}000\varepsilon}{1 + 300\varepsilon} \quad 0 \leq \varepsilon \leq 0.03 \ (\sigma = \text{ksi}) \quad \text{(Eq. 1)}$$

(a) STRESS-STRAIN DIAGRAM (From Eq. 1)



INITIAL SLOPE OF STRESS-STRAIN CURVE

Take the derivative of σ with respect to ε:

$$\frac{d\sigma}{d\varepsilon} = \frac{(1 + 300\varepsilon)(18{,}000) - (18{,}000)(300)\sigma}{(1 + 300\varepsilon)^2}$$

$$= \frac{18{,}000}{(1 + 300\varepsilon)^2}$$

At $\varepsilon = 0$, $\quad \dfrac{d\sigma}{d\varepsilon} = 18{,}00\text{ ksi}$

$\therefore$ Initial slope $= 18{,}000$ ksi

ALTERNATIVE FORM OF THE STRESS-STRAIN RELATIONSHIP

Solve Eq. (1) for ε in terms of σ:

$$\varepsilon = \frac{\sigma}{18{,}000 - 300\sigma} \quad 0 \leq \sigma \leq 54\text{ ksi} \ (\sigma = \text{ksi}) \quad \text{(Eq. 2)}$$

This equation may also be used when plotting the stress-strain diagram.

(b) ELONGATION δ OF THE WIRE

$$\sigma = \frac{P}{A} = \frac{600\text{ lb}}{\frac{\pi}{4}(0.125\text{ in.})^2} 48{,}900\text{ psi} = 48.9\text{ ksi}$$

From Eq. (2) or from the stress-strain diagram:

$\varepsilon = 0.0147$

$\delta = \varepsilon L = (0.0147)(48\text{ in.}) = 0.71\text{ in.} \quad \leftarrow$

STRESS AND STRAIN AT POINT B (see diagram)

$\sigma_B = 48.9\text{ ksi} \quad \varepsilon_B = 0.0147$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{48.9\text{ ksi}}{18{,}000\text{ ksi}} = 0.00272$$

RESIDUAL STRAIN ε_R

$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.0147 - 0.0027 = 0.0120$

(c) Permanent set $= \varepsilon_R L = (0.0120)(48\text{ in.})$

$= 0.58\text{ in.} \quad \leftarrow$

(d) Proportional limit when reloaded $= \sigma_B$

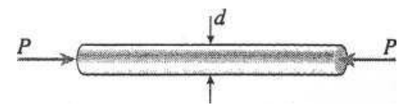
$\sigma_B = 49\text{ ksi} \quad \leftarrow$

Linear Elasticity, Hooke's Law, and Poisson's Ratio

When solving the problems for Section 1.5, assume that the material behaves linearly elastically.

Problem 1.6-1 A high-strength steel bar used in a large crane has diameter $d = 2.00$ in. (see figure). The steel has modulus of elasticity $E = 29 \times 10^6$ psi and Poisson's ratio $\nu = 0.29$. Because of clearance requirements, the diameter of the bar is limited to 2.001 in. when it is compressed by axial forces.

What is the largest compressive load P_{max} that is permitted?



Solution 1.6-1 Steel bar in compression

STEEL BAR

$d = 2.00$ in. Maximum $\Delta d = 0.001$ in.

$E = 29 \times 10^6$ psi $\nu = 0.29$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{0.001 \text{ in.}}{2.00 \text{ in.}} = 0.0005$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = -\frac{0.0005}{0.29} = -0.001724$$

(shortening)

AXIAL STRESS

$$\sigma = E\varepsilon = (29 \times 10^6 \text{ psi})(-0.001724)$$
$$= -50.00 \text{ ksi (compression)}$$

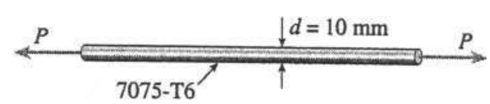
Assume that the yield stress for the high-strength steel is greater than 50 ksi. Therefore, Hooke's law is valid.

MAXIMUM COMPRESSIVE LOAD

$$P_{max} = \sigma A = (50.00 \text{ ksi})\left(\frac{\pi}{4}\right)(2.00 \text{ in.})^2$$
$$= 157 \text{ k} \quad \leftarrow$$

Problem 1.6-2 A round bar of 10 mm diameter is made of aluminum alloy 7075-T6 (see figure). When the bar is stretched by axial forces P, its diameter decreases by 0.016 mm.

Find the magnitude of the load P. (Obtain the material properties from Appendix I.)



Solution 1.6-2 Aluminum bar in tension

$d = 10$ mm $\Delta d = 0.016$ mm

(Decrease in diameter)

7075-T6

From Table I-2: $E = 72$ GPa $\nu = 0.33$

From Table I-3: Yield stress $\sigma_Y = 480$ MPa

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{-0.016 \text{ mm}}{10 \text{ mm}} = -0.0016$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = \frac{0.0016}{0.33}$$
$$= 0.004848 \text{ (Elongation)}$$

AXIAL STRESS

$$\sigma = E\varepsilon = (72 \text{ GPa})(0.004848)$$
$$= 349.1 \text{ MPa (Tension)}$$

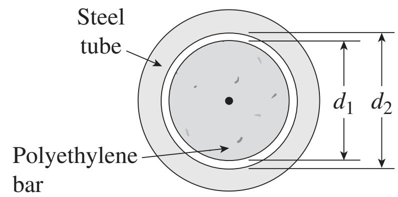
Because $\sigma < \sigma_Y$, Hooke's law is valid.

LOAD P (TENSILE FORCE)

$$P = \sigma A = (349.1 \text{ MPa})\left(\frac{\pi}{4}\right)(10 \text{ mm})^2$$
$$= 27.4 \text{ kN} \quad \leftarrow$$

Problem 1.6-3 A polyethylene bar having diameter $d_1 = 4.0$ in. is placed inside a steel tube having inner diameter $d_2 = 4.01$ in. (see figure). The polyethylene bar is then compressed by an axial force P.

At what value of the force P will the space between the nylon bar and the steel tube be closed? (For nylon, assume $E = 400$ ksi and $v = 0.4$.)



Solution 1.6-3

NUMERICAL DATA

$d_1 = 4$ in. $\quad d_2 = 4.01$ in. $\quad E = 200$ ksi

$v = 0.4 \quad \Delta d_1 = 0.01$ in.

$$A_1 = \frac{\pi}{4}d_1^{\,2} \qquad A_2 = \frac{\pi}{4}d_2^{\,2} \qquad A_1 = 12.566 \text{ in.}^2$$

$A_2 = 12.629$ in.^{2}

LATERAL STRAIN

$$\varepsilon_p = \frac{\Delta d_1}{d_1} \qquad \varepsilon_p = \frac{0.01}{4} \qquad \varepsilon_p = 2.5 \times 10^{-3}$$

NORMAL STRAIN

$$\varepsilon_1 = \frac{-\varepsilon_p}{v} \qquad \varepsilon_1 = -6.25 \times 10^{-3}$$

AXIAL STRESS

$\sigma_1 = E\,\varepsilon_1 \qquad \sigma_1 = -1.25$ ksi

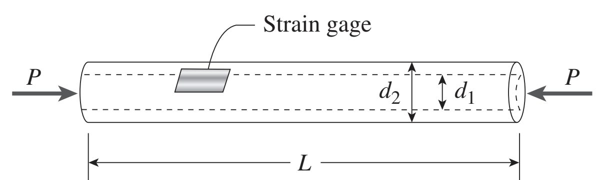
COMPRESSION FORCE

$P = EA_1\varepsilon_1$

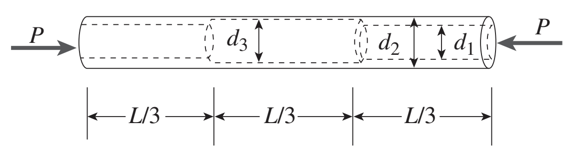
$P = -15.71$ kips $\leftarrow$

Problem 1.6-4 A circular aluminum tube of length $L = 600$ mm is loaded in compression by forces P (see figure). The outside and inside diameters are $d_2 = 75$ mm and d_1= 63 mm, respectively. A strain gage is placed on the outside of the tube to measure normal strains in the longitudinal direction. Assume that $E = 73$ GPa and Poisson's ration $v = 0.33$.

(a) If the compressive stress in the tube is 57 MPa, what is the load P?
(b) If the measured strain is $\varepsilon = 781 \times 10^{-6}$, what is the shortening δ of the tube? What is the percent change in its cross-sectional area? What is the volume change of the tube?
(c) If the tube has a constant outer diameter of $d_2 = 75$ mm along is entire length L but *now* has increased *inner* diameter d_3 with a normal stress of 70 MPa over the middle third, while the rest of the tube remains at normal stress of 57 MPa, what is the diameter d_3?



(a)



(b)

Solution 1.6-4

(a) GIVEN STRESS, FIND FORCE P IN BAR FIGURE (A)

$\sigma = 57 \text{ MPa} \quad E = 73 \text{ GPa} \quad \nu = 0.33$

$L = 600 \text{ mm} \quad d_2 = 75 \text{ mm} \quad d_1 = 63 \text{ mm}$

$$A = \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = 1301 \text{ mm}^2$$

$\boxed{P = \sigma A = 74.1 \text{ kN}}$

(b) GIVEN STRAIN, FIND CHANGE IN LENGTH IN BAR FIGURE (A) AND ALSO VOLUME CHANGE

$$\varepsilon = -781\left(10^{-6}\right) \qquad t = \frac{d_2 - d_1}{2} = 6 \text{ mm}$$

$\boxed{\delta = \varepsilon L = -0.469 \text{ mm}}$ shortening

$$\text{Vol}_1 = L(A) = 7.804 \times 10^5 \text{ mm}^3$$

$$\varepsilon_{lat} = -\nu\varepsilon = 2.577 \times 10^{-4} \qquad \Delta t = \varepsilon_{lat}\, t = 1.546 \times 10^{-3} \text{ mm}$$

$$\Delta d_2 = \varepsilon_{lat} d_2 = 0.019 \text{ mm} \qquad \Delta d_1 = \varepsilon_{lat} d_1 = 0.016 \text{ mm}$$

$$A_f = \frac{\pi}{4}\left[\left(d_2 + \Delta d_2\right)^2 - \left(d_1 + \Delta d_1\right)^2\right]$$

$$A_f = 1301.29 \text{ mm}^2$$

$$\frac{A_f - A}{A} = \frac{1301.29 - 1300.62}{1300.62} = 0.052\%$$

$$V_{1f} = (L + \delta)\left(A_f\right) = 7.802 \times 10^5 \text{ mm}^3$$

$$\Delta V_1 = V_{1f} - \text{Vol}_1 = -207.482 \text{ mm}^3$$

$\boxed{\Delta V_1 = -207 \text{ mm}^3}$ change

(c) IF THE TUBE HAS CONSTANT OUTER DIAMETER OF $d_2 = 75$ mm ALONG ITS ENTIRE LENGTH L BUT NOW HAS INCREASED INNER DIAMETER d_3 OVER THE MIDDLE THIRD WITH NORMAL STRESS OF 70 MPa, WHILE THE REST OF THE BAR REMAINS AT NORMAL STRESS OF 57 MPa, WHAT IS THE DIAMETER d_3?

$$\sigma_{M3} = 70 \text{ MPa} \quad P = 74.135 \text{ kN} \quad A_{M3} = \frac{P}{\sigma_{M3}} = 1059.076 \text{ mm}^2 \qquad d_2 = 75 \text{ mm} \qquad d_1 = 63 \text{ mm}$$

$$d_2^2 - d_3^2 = \frac{4}{\pi}A_{M3} \qquad \text{SO} \qquad d_3 = \sqrt{d_2^2 - \frac{4}{\pi}A_{M3}} = 65.4 \text{ mm} \qquad t_{M3} = \frac{d_2 - d_3}{2} = 4.802 \text{ mm}$$

$\boxed{d_3 = 65.4 \text{ mm}}$

Problem 1.6-5 A bar of monel metal as in the figure (length $L = 9$ in., diameter $d = 0.225$ in.) is loaded axially by a tensile force P. If the bar elongates by 0.0195 in., what is the decrease in diameter d? What is the magnitude of the load P? Use the data in Table I-2, Appendix I.

Solution 1.6-5

NUMERICAL DATA

$E = 25{,}000$ ksi

$\nu = 0.32$

$L = 9$ in.

$\delta = 0.0195$ in.

$d = 0.225$ in.

NORMAL STRAIN

$$\varepsilon = \frac{\delta}{L} \quad \varepsilon = 2.167 \times 10^{-3}$$

LATERAL STRAIN

$$\varepsilon_p = -\nu\varepsilon \quad \varepsilon_p = -6.933 \times 10^{-4}$$

DECREASE IN DIAMETER

$\Delta d = \varepsilon_p d$

$\Delta d = -1.56 \times 10^{-4}$ in. $\leftarrow$

INITIAL CROSS SECTIONAL AREA

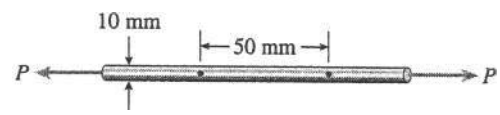
$$A_i = \frac{\pi}{4}d^2 \quad A_i = 0.04 \text{ in.}^2$$

MAGNITUDE OF LOAD P

$P = EA_i\varepsilon$

$P = 2.15$ kips $\leftarrow$

Problem 1.6-6 A tensile test is peformed on a brass specimen 10 mm in diameter using a gage length of 50 mm (see figure). When the tensile load P reaches a value of 20 kN, the distance between the gage marks has increased by 0.122 mm.



(a) What is the modulus of elasticity E of the brass?
(b) If the diameter decreases by 0.00830 mm, what is Poisson's ratio?

Solution 1.6-6 Brass specimen in tension

$d = 10$ mm Gage length $L = 50$ mm

$P = 20$ kN $\delta = 0.122$ mm $\Delta d = 0.00830$ mm

AXIAL STRESS

$$\sigma = \frac{P}{A} = \frac{20 \text{ k}}{\frac{\pi}{4}(10 \text{ mm})^2} = 254.6 \text{ MPa}$$

Assume σ is below the proportional limit so that Hooke's law is valid.

AXIAL STRAIN

$$\varepsilon = \frac{\delta}{L} = \frac{0.122 \text{ mm}}{50 \text{ mm}} = 0.002440$$

(a) MODULUS OF ELASTICITY

$$E = \frac{\sigma}{\varepsilon} = \frac{254.6 \text{ MPa}}{0.002440} = 104 \text{ GPa} \quad \leftarrow$$

(b) POISSON'S RATIO

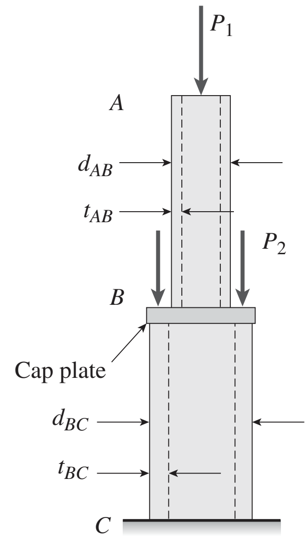
$\varepsilon' = \nu\varepsilon$

$\Delta d = \varepsilon' d = \nu\varepsilon d$

$$\nu = \frac{\Delta d}{\varepsilon d} = \frac{0.00830 \text{ mm}}{(0.002440)(10 \text{ mm})} = 0.34 \quad \leftarrow$$

Problem 1.6-7 A hollow, brass circular pipe ABC (see figure) supports a load $P_1 = 26.5$ kips acting at the top. A second load $P_2 = 22.0$ kips is uniformly distributed around the cap plate at B. The diameters and thicknesses of the upper and lower parts of the pipe are $d_{AB} = 1.25$ in., $t_{AB} = 0.5$ in., $d_{BC} = 2.25$ in., and $t_{AB} = 0.375$ in., respectively. The modulus of elasticity is 14,000 ksi. When both loads are fully applied, the wall thickness of pipe BC increases by 200×10^{-6} in.

(a) Find the increase in the inner diameter of pipe segment BC.
(b) Find Poisson's ratio for the brass.
(c) Find the increase in the wall thickness of pipe segment AB and the increase in the inner diameter of AB.



Solution 1.6-7

NUMERICAL DATA

$P_1 = 26.5$ k

$P_2 = 22$ k

$d_{AB} = 1.25$ in.

$t_{AB} = 0.5$ in.

$d_{BC} = 2.25$ in.

$t_{BC} = 0.375$ in.

$E = 14000$ ksi

$\Delta t_{BC} = 200 \times 10^{-6}$

(a) INCREASE IN THE INNER DIAMETER OF PIPE SEGMENT BC

$$\varepsilon_{pBC} = \frac{\Delta t_{BC}}{t_{BC}} \qquad \varepsilon_{pBC} = 5.333 \times 10^{-4}$$

$$\Delta d_{BC\text{inner}} = \varepsilon_{pBC}(d_{BC} - 2t_{BC})$$

$$\Delta\, d_{BC\text{inner}} = 8 \times 10^{-4} \text{ in.} \quad \leftarrow$$

(b) POISSON'S RATIO FOR THE BRASS

$$A_{BC} = \frac{\pi}{4}\left[d_{BC}{}^2 - (d_{BC} - 2t_{BC})^2\right]$$

$$A_{BC} = 2.209 \text{ in.}^2$$

$$\varepsilon_{BC} = \frac{-(P_1 + P_2)}{(EA_{BC})} \qquad \varepsilon_{BC} = -1.568 \times 10^{-3}$$

$$\nu_{\text{brass}} = \frac{-\varepsilon_{pBC}}{\varepsilon_{BC}} \qquad \nu_{\text{brass}} = 0.34$$

(agrees with App. I (Table I-2))

(c) INCREASE IN THE WALL THICKNESS OF PIPE SEGMENT AB AND THE INCREASE IN THE INNER DIAMETER OF AB

$$A_{AB} = \frac{\pi}{4}\left[d_{AB}{}^2 - (d_{AB} - 2t_{AB})^2\right]$$

$$\varepsilon_{AB} = \frac{-P_1}{EA_{AB}} \qquad \varepsilon_{AB} = -1.607 \times 10^{-3}$$

$$\varepsilon_{pAB} = -\nu_{\text{brass}}\varepsilon_{AB} \qquad \varepsilon_{pAB} = 5.464 \times 10^{-4}$$

$$\Delta t_{AB} = \varepsilon_{pAB} t_{AB} \qquad \Delta t_{AB} = 2.73 \times 10^{-4} \text{ in.} \quad \leftarrow$$

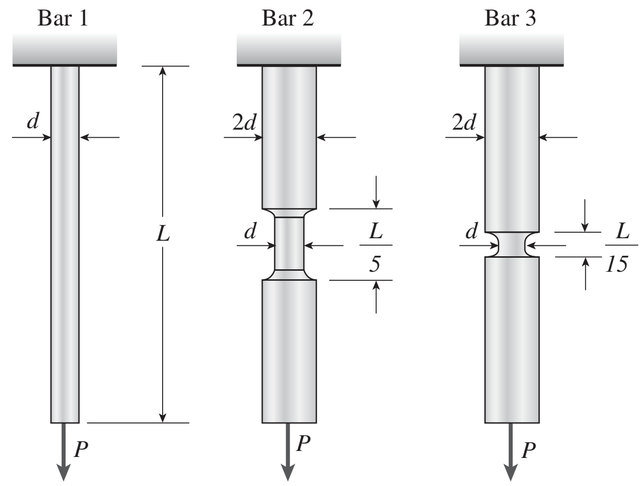
$$\Delta d_{AB\text{inner}} = \varepsilon_{pAB}(d_{AB} - 2t_{AB})$$

$$\Delta d_{AB\text{inner}} = 1.366 \times 10^{-4} \text{ in.}$$

Problem 1.6-8 Three round, copper alloy bars having the same length L but different shapes are shown in the figure. The first bar has a diameter d over is entire length, the second has a diameter d over one-fifth of is length, and the third has a diameter d over one-fifteenth of its length. Elsewhere, the second and third bars have diameter $2d$. All three bars are subjected to the same axial load P.

Use the following numerical date: $P = 1400$ kN, $L = 5$ m, $d = 80$ mm, $E = 110$ GPa, and $\nu = 0.33$.

(a) Find the change in length of each bar.
(b) Find the change in volume of each bar.



Solution 1.6-8

$P = 1400$ kN $\quad L = 5$ m $\quad d = 80$ mm $\quad E = 110$ GPa $\quad \nu = 0.33$

$$A_d = \frac{\pi}{4}d^2 = 5026.5 \text{ mm}^2 \qquad A_{2d} = \frac{\pi}{4}(2\,d)^2 = 20{,}106.2 \text{ mm}^2$$

(a) FIND CHANGE IN LENGTH OF EACH BAR

BAR #1 $\quad \varepsilon_1 = \dfrac{P}{EA_d} = 2.532 \times 10^{-3} \qquad \sigma_1 = E\varepsilon_1 = 279 \text{ MP}a$

< Appendix I, Table I-3: copper alloys can have yield stress in range 55–760 MPa so assume this is below proportional limit so that Hooke's Law applies

$$\boxed{\Delta L_1 = \varepsilon_1 L = 12.66 \text{ mm}} \qquad L_{f1} = L + \Delta L_1 = 5012.66 \text{ mm}$$

BAR #2 $\quad \varepsilon_{2a} = \dfrac{P}{EA_d} = 2.532 \times 10^{-3} \qquad \varepsilon_{2b} = \dfrac{P}{EA_{2d}} = 6.33 \times 10^{-4} \qquad \dfrac{\varepsilon_{2a}}{4} = 6.33 \times 10^{-4}$

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{5} = 2.532 \text{ mm} \qquad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{4L}{5}\right) = 2.532 \text{ mm}$$

$$\boxed{\Delta L_2 = \Delta L_{2a} + \Delta L_{2b} = 5.06 \text{ mm}} \qquad L_{f2} = L + \Delta L_2 = 5005.06 \text{ mm} \qquad \frac{\Delta L_2}{\Delta L_1} = 0.4$$

BAR #3

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{15} = 0.844 \text{ mm} \qquad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{14L}{15}\right) = 2.954 \text{ mm}$$

$$\boxed{\Delta L_3 = \Delta L_{2a} + \Delta L_{2b} = 3.8 \text{ mm}} \qquad L_{f3} = L + \Delta L_3 = 5003.08 \text{ mm} \qquad \frac{\Delta L_3}{\Delta L_1} = 0.3$$

(b) FIND CHANGE IN VOLUME OF EACH BAR

Use lateral strain (ε_p) in each segment to find change in diameter Δd, then find change in cross sectional area, then volume

BAR #1

$$\varepsilon_{p1} = -\nu\varepsilon_1 = -8.356 \times 10^{-4} \qquad \Delta d_1 = \varepsilon_{p1}d = -0.067 \text{ mm} \qquad A_1 = \frac{\pi}{4}(d + \Delta d_1)^2 = 5018.152 \text{ mm}^2$$

$$\Delta \text{Vol}_1 = A_1 L_{f1} - A_d L = 21548 \text{ mm}^3 \qquad \frac{\Delta Vol_1}{A_d L} = 8.574 \times 10^{-4}$$

Bar #2

$$\varepsilon_{p2a} = \varepsilon_{p1} \quad \varepsilon_{p2b} = -\nu\,\varepsilon_{2b} = -2.089 \times 10^{-4} \quad \frac{\varepsilon_{p1}}{4} = -2.089 \times 10^{-4}$$

$$\Delta d_{2b} = \varepsilon_{p2b}(2d) = -0.33\text{ mm} \quad A_{2a} = A_1 \quad A_{2b} = \frac{\pi}{4}(2d + \Delta d_{2b})^2 = 20097.794\text{ mm}^2$$

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{5} = 2.532\text{ mm} \quad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{4L}{5}\right) = 2.532\text{ mm}$$

$$\Delta\text{Vol}_2 = \left[A_1\left(\frac{L}{5} + \Delta L_{2a}\right) + A_{2b}\left(\frac{4L}{5} + \Delta L_{2b}\right)\right] - \left[A_{2d}\left(\frac{4L}{5}\right) + A_d\left(\frac{L}{5}\right)\right]$$

$$= 21601\text{ mm}^3 \qquad \frac{\Delta\text{Vol}_2}{\Delta\text{Vol}_1} = 1.002$$

Bar #3

$$\Delta L_{2a} = \varepsilon_{2a}\frac{L}{15} = 0.844\text{ mm} \qquad \Delta L_{2b} = \varepsilon_{2b}\left(\frac{14L}{15}\right) = 2.954\text{ mm}$$

$$\Delta\text{Vol}_3 = \left[A_1\left(\frac{L}{15} + \Delta L_{2a}\right) + A_{2b}\left(\frac{14L}{15} + \Delta L_{2b}\right)\right] - \left[A_{2d}\left(\frac{14L}{15}\right) + A_d\left(\frac{L}{15}\right)\right]$$

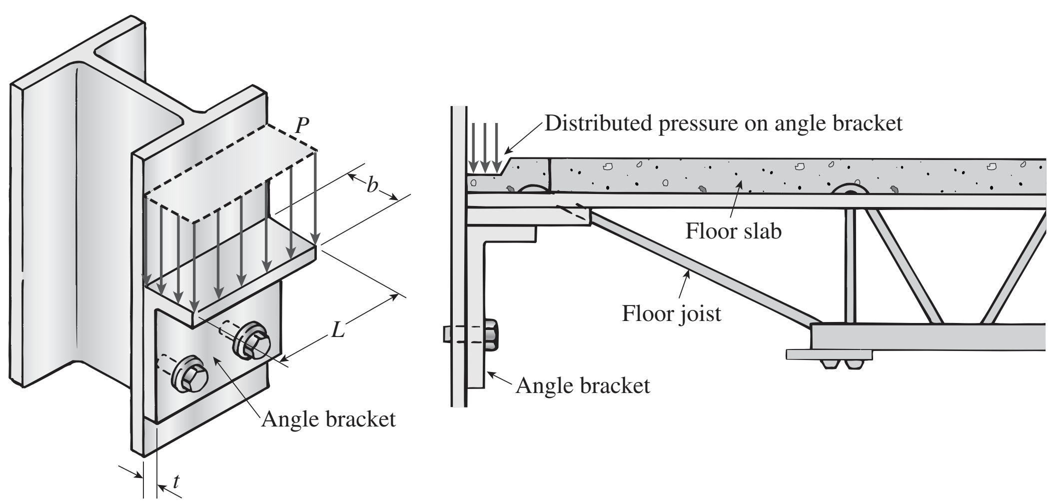
$$= 21610\text{ mm}^3 \qquad \frac{\Delta\text{Vol}_3}{\Delta\text{Vol}_2} = 1.003$$

$$\boxed{\Delta\text{Vol}_1 = 21548\text{ mm}^3} \quad \boxed{\Delta\text{Vol}_2 = 21601\text{ mm}^3} \quad \boxed{\Delta\text{Vol}_3 = 21610\text{ mm}^3}$$

Shear Stress and Strain

Problem 1.7-1 An angle bracket having thickness $t = 0.75$ in. is attached to the flange of a column by two 5/8-inch diameter bolts (see figure). A uniformly distributed load from a floor joist acts on the top face of the bracket with a pressure $p = 275$ psi. The top face of the bracket has length $L = 8$ in. and width $b = 3.0$ in.

Determine the average bearing pressure σ_b between the angle bracket and the bolts and the average shear stress τ_{aver} in the bolts. (Disregard friction between the bracket and the column.)



Solution 1.7-1

Numerical data

$t = 0.75$ in. $\quad L = 8$ in.

$b = 3.$ in. $\quad p = \dfrac{275}{1000}$ ksi $\quad d = \dfrac{5}{8}$ in.

Bearing force

$F = pbL \qquad F = 6.6$ k

Shear and bearing areas

$A_S = \dfrac{\pi}{4}d^2 \qquad A_S = 0.307\text{ in.}^2$

$A_b = dt \qquad A_b = 0.469\text{ in.}^2$

Bearing stress

$\sigma_b = \dfrac{F}{2A_b} \qquad \sigma_b = 7.04$ ksi $\leftarrow$

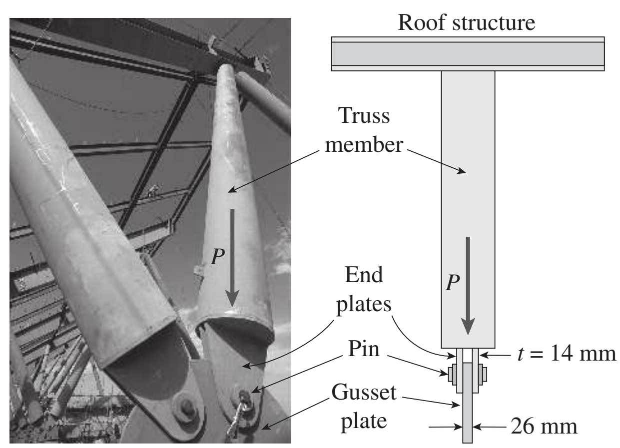
Shear stress

$\tau_{\text{ave}} = \dfrac{F}{2A_S} \qquad \tau_{\text{ave}} = 10.76$ ksi $\leftarrow$

Problem 1.7-2 Truss members supporting a roof are connected to a 26-mm-thick gusset plate by a 22-mm diameter pin as shown in the figure and photo. The two end plates on the truss members are each 14 mm thick.

(a) If the load $P = 80$ kN, what is the largest bearing stress acting on the pin?
(b) If the ultimate shear stress for the pin is 190 MPa, what force P_{ult} is required to cause the pin to fail in shear?

(Disregard friction between the plates.)



Solution 1.7-2

Numerical data

$t_{ep} = 14$ mm

$t_{gp} = 26$ mm

$P = 80$ kN

$d_p = 22$ mm

$\tau_{\text{ult}} = 190$ MPa

(a) Bearing stress on pin

$\sigma_b = \dfrac{P}{d_p t_{gp}}$ gusset plate is thinner than ($2\,t_{ep}$) so gusset plate controls

$\sigma_b = 139.9$ MPa $\leftarrow$

(b) Ultimate force in shear

Cross sectional area of pin

$A_p = \dfrac{\pi d_p^2}{4}$

$A_p = 380.133\text{ mm}^2$

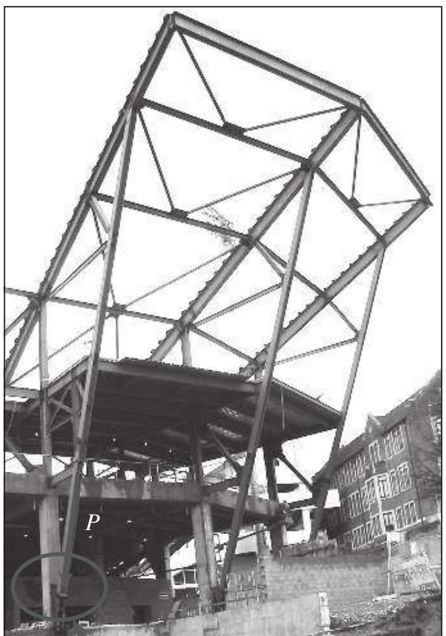
$P_{\text{ult}} = 2\tau_{\text{ult}}A_p \qquad P_{\text{ult}} = 144.4$ kN $\leftarrow$

Problem 1.7-3 The upper deck of a football stadium is supported by braces each of which transfers a load $P = 160$ kips to the base of a column [see figure part (a)]. A cap plate at the bottom of the brace distributes the load P to four flange plates ($t_f = 1$ in.) through a pin ($d_p = 2$ in.) to two gusset plates ($t_g = 1.5$ in.) [see figure parts (b) and (c)].

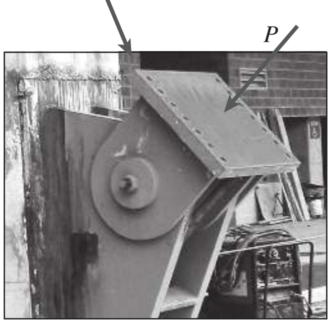
Determine the following quantities.

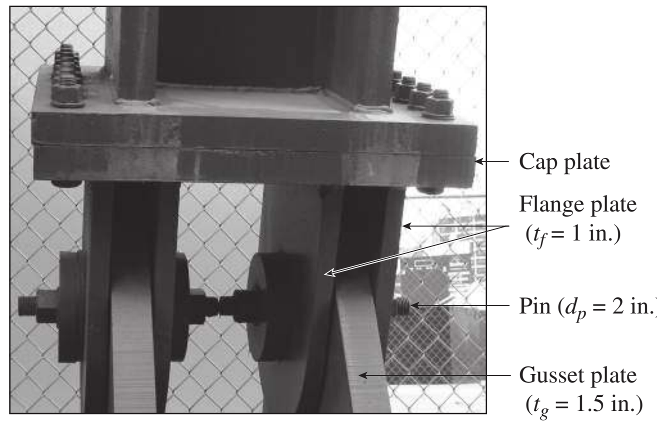
(a) The average shear stress τ_{aver} in the pin.
(b) The average bearing stress between the flange plates and the pin (σ_{bf}), and also between the gusset plates and the pin (σ_{bg}).

(Disregard friction between the plates.)

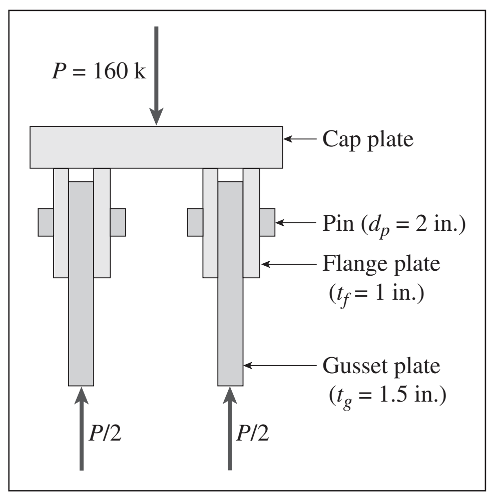


(a) Stadium brace





(b) Detail at bottom of brace



(c) Section through bottom of brace

Solution 1.7-3

NUMERICAL DATA

$P = 160$ kips $\quad d_p = 2$ in.

$t_g = 1.5$ in. $\quad t_f = 1$ in.

(a) SHEAR STRESS ON PIN

$$\tau = \frac{V}{\left(\frac{\pi d_p^2}{4}\right)} \qquad \tau = \frac{\frac{P}{4}}{\left(\frac{\pi d_p^2}{4}\right)}$$

$\tau = 12.73$ ksi $\leftarrow$

(b) BEARING STRESS ON PIN FROM FLANGE PLATE

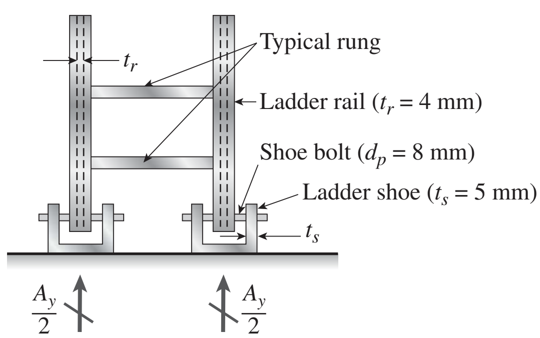
$$\sigma_{bf} = \frac{\frac{P}{4}}{d_p t_f} \qquad \sigma_{bf} = 20 \text{ ksi} \quad \leftarrow$$

BEARING STRESS ON PIN FROM GUSSET PLATE

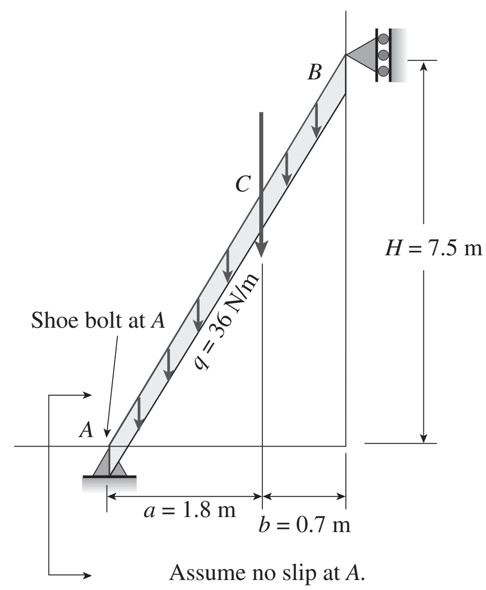
$$\sigma_{bg} = \frac{\frac{P}{2}}{d_p t_g} \qquad \sigma_{bg} = 26.7 \text{ ksi} \quad \leftarrow$$

Problem 1.7-4 The inclined ladder AB supports a house painter (85 kg) at C and the self weight ($q = 40$ N/m) of the ladder itself. Each ladder rail ($t_r = 4$ mm) is supported by a shoe ($t_s = 5$mm) which is attached to the ladder rail by a bolt of diameter $d_p = 8$ mm.

(a) Find support reactions at A and B
(b) Find the resultant force in the shoe bolt at A.
(c) Find maximum average shear (τ) and bearing (σ_b) stresses in the shoe bolt at A.



Section at base



Assume no slip at A.

Solution 1.7-4

NUMERICAL DATA $\quad t_r = 4$ mm $\quad t_s = 5$ mm $\quad d_p = 8$ mm $\quad P = 85(9.81) \quad P = 833.85$ N

$a = 1.8$ m $\quad b = 0.7$ m $\quad H = 7.5$ m $\quad q = 40$ N/m

(a) SUPPORT REACTIONS

$$L = \sqrt{(a + b)^2 + H^2} \qquad L = 7.906 \text{ m} \qquad L_{AC} = \frac{a}{a + b} L \quad L_{AC} = 5.692$$

$$L_{CB} = \frac{a}{a + b} L \quad L_{CB} = 2.214 \qquad L_{AC} + L_{CB} = 7.906$$

Sum moments about A

$$B_x = \frac{Pa + qL\left(\dfrac{a+b}{2}\right)}{-H} \qquad B_x = -252.829 \text{ N (left) and } A_x = -B_x (A_x \text{ acts to right}) \qquad A_x = -B_x$$

$$A_y = P + qL \qquad A_y = 1150.078 \text{ N} \qquad \boxed{B_x = -252.8 \text{ N}} \quad \boxed{A_x = -B_x} \quad \boxed{A_y = 1150.1 \text{ N}}$$

(b) Resultant force in shoe bolt at A $\qquad A_{\text{resultant}} = \sqrt{A_x^2 + A_y^2}$

$$A_{\text{resultant}} = 1177.54 \text{ N} \qquad \boxed{A_{\text{resultant}} = 1178 \text{ N}}$$

(c) Maximum shear and bearing stresses in shoe bolt at A

$$d_p = 8 \text{ mm} \qquad t_s = 5 \text{ mm} \qquad t_r = 4 \text{ mm}$$

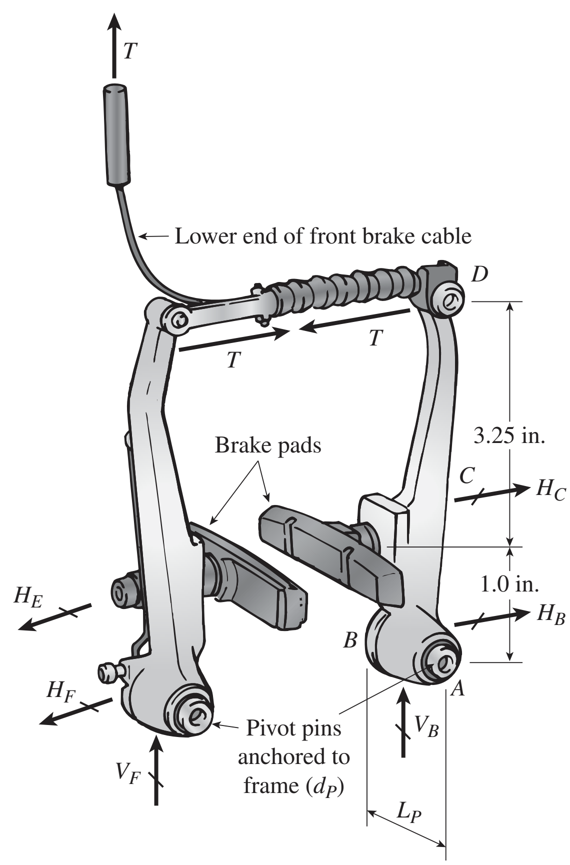
Shear area: $A_s = \frac{\pi}{4}d_p^2 \quad A_s = 50.256 \text{ mm}^2$ Shear stress: $\tau = \dfrac{\dfrac{A_{\text{resultant}}}{2}}{2A_s}$ $\boxed{\tau = 5.86 \text{ MPa}}$

Bearing area: $A_b = 2d_pt_s \quad A_b = 80 \text{ mm}^2$ Bearing stress: $\sigma_{b\text{shore}} = \dfrac{\dfrac{A_{\text{resultant}}}{2}}{A_b}$ $\boxed{\sigma_{b\text{shoe}} = 7.36 \text{ MPa}}$

Problem 1.7-5 The force in the brake cable of the V-brake system shown in the figure is $T = 45$ lb. The pivot pin at A has diameter $d_p = 0.25$ in. and length $L_p = 5/8$ in.

Use dimensions show in the figure. Neglect the weight of the brake system.

(a) Find the average shear stress τ_{aver} in the pivot pin where it is anchored to the bicycle frame at B.
(b) Find the average bearing stress $\sigma_{b,\text{aver}}$ in the pivot pin over segment AB.



Solution 1.7-5

NUMERICAL DATA

$d_p = 0.25$ in. $\quad L = \dfrac{5}{8}$ in. $\quad CD = 3.25$ in.

$BC = 1$ in. $\quad T = 45$ lb

EQUILIBRIUM - FIND HORIZONTAL FORCES AT B AND C [VERTICAL REACTION $V_B = 0$]

$\sum M_B = 0 \qquad H_C = \dfrac{T(BC + CD)}{BC}$

$H_C = 191.25$ lb $\qquad \sum F_H = 0$

$H_B = T - H_C \qquad H_B = -146.25$ lb

(a) FIND THE AVE SHEAR STRESS τ_{ave} IN THE PIVOT PIN WHERE IT IS ANCHORED TO THE BICYCLE FRAME AT B:

$A_S = \dfrac{\pi d_p{}^2}{4} \qquad A_s = 0.049 \text{ in.}^2$

$\tau_{ave} = \dfrac{|H_B|}{A_S} \qquad \tau_{ave} = 2979 \text{ psi} \quad \leftarrow$

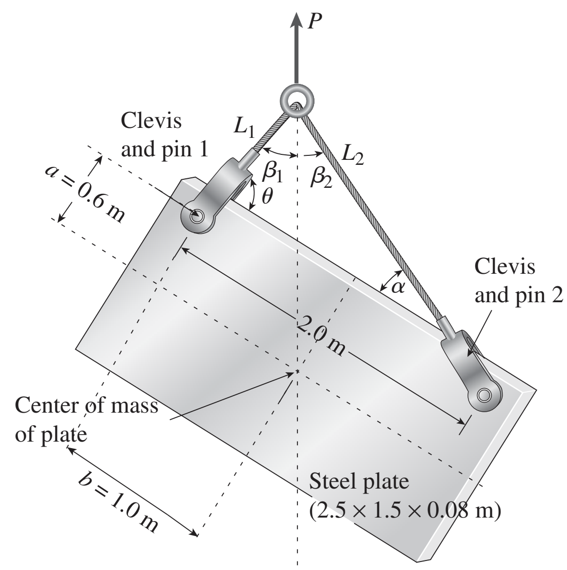
(b) FIND THE AVE BEARING STRESS $\sigma_{b,ave}$ IN THE PIVOT PIN OVER SEGMENT AB.

$A_b = d_p L \qquad A_b = 0.156 \text{ in.}^2$

$\sigma_{b,ave} = \dfrac{|H_B|}{A_b} \qquad \sigma_{b,ave} = 936 \text{ psi} \quad \leftarrow$

Problem 1.7-6 A steel plate of dimensions $2.5 \times 1.5 \times 0.08$ m and weighing 23.1kN is hoisted by steel cables with lengths $L_1 = 3.2$ m and $L_2 = 3.9$ m that are each attached to the plate by a clevis and pin (see figure). The pins through the clevises are 18 mm in diameter and are located 2.0 m apart. The orientation angles are measured to be $\theta = 94.4°$ and $\alpha = 54.9°$.

For these conditions, first determine the cable forces T_1 and T_2, then find the average shear stress τ_{aver} in both pin 1 and pin 2, and then the average bearing stress σ_b between the steel plate and each pin. Ignore the mass of the cables.



Solution 1.7-6

NUMERICAL DATA

$L_1 = 3.2$ m $\quad L_2 = 3.9$ m $\quad \alpha = 54.9\left(\dfrac{\pi}{180}\right)$ rad

$\theta = 94.4\left(\dfrac{\pi}{180}\right)$ rad

$a = 0.6$ m $\qquad b = 1$ m

$W = 77.0(2.5 \times 1.5 \times 0.08) \qquad W = 23.1$ kN

($77 =$ wt density of steel, kN/m^3)

SOLUTION APPROACH

STEP (1) $d = \sqrt{a^2 + b^2} \qquad d = 1.166$ m

STEP (2) $\theta_1 = \arctan\left(\dfrac{a}{b}\right) \qquad \theta_1 \dfrac{180}{\pi} = 30.964°$

STEP (3)-Law of cosines

$$H = \sqrt{d^2 + L_1^2 - 2dL_1\cos(\theta + \theta_1)}$$

$H = 3.99$ m

STEP (4) $\beta_1 = \arccos\left(\frac{L_2^2 + H^2 - d^2}{2L_1H}\right)$

$$\beta_1\frac{180}{\pi} = 13.789°$$

STEP (5) $\beta_2 = \arccos\left(\frac{L_2^2 + H^2 - d^2}{2L_2H}\right)$

$$\beta_2\frac{180}{\pi} = 16.95°$$

STEP (6)

Check $(\beta_1 + \beta_2 + \theta + \alpha)\frac{180}{\pi}$

$= 180.039°$

STATICS

$T_1\sin(\beta_1) = T_2\sin(\beta_2)$

$$T_1 = T_2\left(\frac{\sin(\beta_2)}{\sin(\beta_1)}\right)$$

$T_1\cos(\beta_1) + T_2\cos(\beta_2) = W$

$$T_2 = \frac{W}{\cos(\beta_1)\frac{\sin(\beta_2)}{\sin(\beta_1)} + \cos(\beta_2)}$$

$T_2 = 10.77\text{ kN} \quad \leftarrow$

$$T_1 = T_2\left(\frac{\sin(\beta_2)}{\sin(\beta_1)}\right) \qquad T_1 = 13.18\text{ kN} \quad \leftarrow$$

$T_1\cos(\beta_1) + T_2\cos(\beta_2) = 23.1 \;<$ checks

SHEAR & BEARING STRESSES

$d_p = 18\text{ mm} \qquad t = 80\text{ mm}$

$$A_S = \frac{\pi}{4}d_p^2 \qquad A_b = td_p$$

$$\tau_{1ave} = \frac{\frac{T_1}{2}}{A_S} \qquad \tau_{1ave} = 25.9\text{ MPa} \quad \leftarrow$$

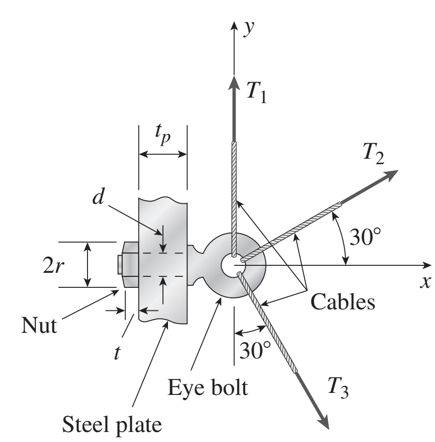
$$\tau_{2ave} = \frac{\frac{T_2}{2}}{A_S} \qquad \tau_{2ave} = 21.2\text{ MPa} \quad \leftarrow$$

$$\sigma_{b1} = \frac{T_1}{A_b} \qquad \sigma_{b1} = 9.15\text{ MPa} \quad \leftarrow$$

$$\sigma_{b2} = \frac{T_2}{A_b} \qquad \sigma_{b2} = 7.48\text{ MPa} \quad \leftarrow$$

Problem 1.7-7 A special-purpose eye bolt of shank diameter $d = 0.50$ in. passes through a hole in a steel plate of thickness $t_p = 0.75$ in. (see figure) and is secured by a nut with thickness $t = 0.25$ in. The hexagonal nut bears directly against the steel plate. The radius of the circumscribed circle for the hexagon is $r = 0.40$ in. (which means that each side of the hexagon has length 0.40 in.). The tensile forces in three cables attached to the eye bolt are $T_1 = 800$ lb., $T_2 = 550$ lb., and $T_3 = 1241$ lb.

(a) Find the resultant force acting on the eye bolt.
(b) Determine the average bearing stress σ_b between the hexagonal nut on the eye bolt and the plate.
(c) Determine the average shear stress τ_{aver} in the nut and also in the steel plate.



Solution 1.7-7

CABLE FORCES

$T_1 = 800$ lb $T_2 = 550$ lb $T_3 = 1241$ lb

(a) RESULTANT

$$P = T_2\frac{\sqrt{3}}{2} + T_3 0.5 \qquad P = 1097 \text{ lb} \quad \leftarrow$$

(b) AVERAGE BEARING STRESS

$A_b = 0.2194$ in.^{2} hexagon (Case 25, Appendix E)

$$\sigma_b = \frac{P}{A_b} \qquad \sigma_b = 4999 \text{ psi} \quad \leftarrow$$

(c) AVERAGE SHEAR THROUGH NUT

$d = 0.5$ in. $t = 0.25$ in.

$$A_{sn} = \pi \text{dt} \qquad A_{sn} = 0 \qquad \tau_{nut} = \frac{P}{A_{sn}}$$

$$\tau_{nut} = 2793 \text{ psi} \quad \leftarrow$$

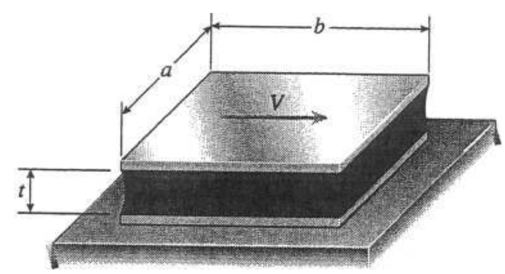
SHEAR THROUGH PLATE $t_p = 0.75$ $r = 0.40$

$$A_{spl} = 6rt_p \qquad A_{spl} = 2$$

$$\tau_{pl} = \frac{P}{A_{spl}} \qquad \tau_{pl} = 609 \text{ psi} \quad \leftarrow$$

Problem 1.7-8 An elastomeric bearing pad consisting of two steel plates bonded to a chloroprene elastomer (an artificial rubber) is subjected to a shear force V during a static loading test (see figure). The pad has dimensions $a = 125$ mm and $b = 240$ mm, and the elastomer has thickness $t = 50$ mm. When the force V equals 12 kN, the top plate is found to have displaced laterally 8.0 mm with respect to the bottom plate.

What is the shear modulus of elasticity G of the chloroprene?

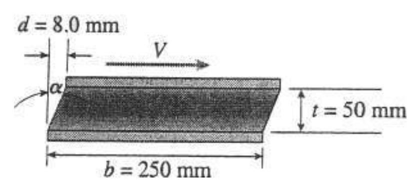


Solution 1.7-8

NUMERICAL DATA

$V = 12$ kN $a = 125$ mm

$b = 240$ mm $t = 50$ mm $d = 8$ mm



AVERAGE SHEAR STRESS

$$\tau_{ave} = \frac{V}{ab} \qquad \tau_{ave} = 0.4 \text{ MPa}$$

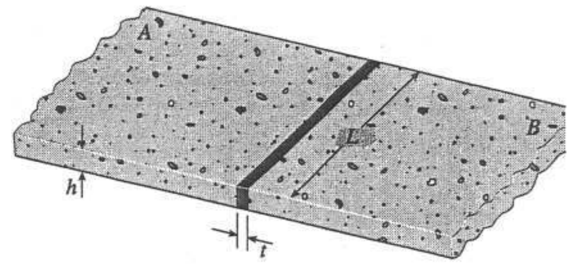
AVERAGE SHEAR STRAIN $\gamma_{ave} = \dfrac{d}{t}$ $\gamma_{ave} = 0.16$

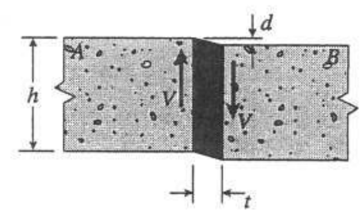
SHEAR MODULUS G $G = \dfrac{\tau_{ave}}{\gamma_{ave}}$

$$G = 2.5 \text{ MPa} \quad \leftarrow$$

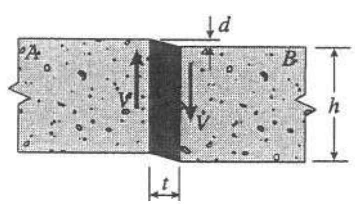
Problem 1.7-9 A joint between two concrete slabs A and B is filled with a flexible epoxy that bonds securely to the concrete (see figure). The height of the joint is $h = 4.0$ in., its length is $L = 40$ in., and its thickness is $t = 0.5$ in. Under the action of shear forces V, the slabs displace vertically through the distance $d = 0.002$ in. relative to each other.

(a) What is the average shear strain γ_{aver} in the epoxy?
(b) What is the magnitude of the forces V if the shear modulus of elasticity G for the epoxy is 140 ksi?





Solution 1.7-9 Epoxy joint between concrete slabs



$h = 4.0$ in. $\quad t = 0.5$ in.

$L = 40$ in. $\quad d = 0.002$ in.

$G = 140$ ksi

(a) Average shear strain

$$\gamma_{aver} = \frac{d}{t} = 0.004 \quad \leftarrow$$

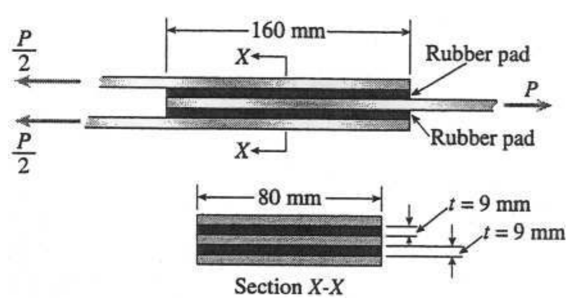
(b) Shear forces V

Average shear stress: $\tau_{aver} = G\gamma_{aver}$

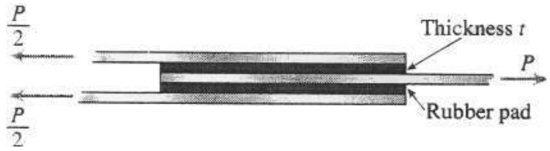
$$V = \tau_{aver}(hL) = G\gamma_{aver}(hL)$$
$$= (140 \text{ ksi})(0.004)(4.0 \text{ in.})(40 \text{ in.})$$
$$= 89.6 \text{ k} \quad \leftarrow$$

Problem 1.7-10 A flexible connection consisting of rubber pads (thickness $t = 9$ mm) bonded to steel plates is shown in the figure. The pads are 160 mm long and 80 mm wide.

(a) Find the average shear strain γ_{aver} in the rubber if the force $P = 16$ kN and the shear modulus for the rubber is $G = 1250$ kPa.
(b) Find the relative horizontal displacement δ between the interior plate and the outer plates.



Solution 1.7-10 Rubber pads bonded to steel plates



Rubber pads: $t = 9$ mm

Length $L = 160$ mm

Width $b = 80$ mm

$G = 1250$ kPa

$P = 16$ kN

(a) Shear stress and strain in the rubber pads

$$\tau_{\text{aver}} = \frac{P/2}{bL} = \frac{8\text{ kN}}{(80\text{ mm})(160\text{ mm})} = 625\text{ kPa}$$

$$\gamma_{\text{aver}} = \frac{\tau_{\text{aver}}}{G} = \frac{625\text{ kPa}}{1250\text{ kPa}} = 0.50 \quad \leftarrow$$

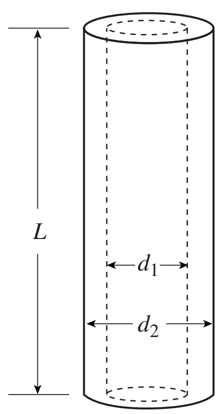
(b) Horizontal displacement

$$\delta = t \times \tan(\gamma_{\text{ave}}) = 4.92\text{ mm}$$

Problem 1.7-11 Steel riser pipe hangs from a drill rig located offshore in deep water (see figure). Separate segments are joined using bolted flange plates (see figure and photo). Assume that there are six bolts at each pipe segment connection. Assume that the total length of riser pipe is $L = 5000$ ft; outer and inner diameters are $d_2 = 16$ in., $d_1 = 15$ in.; flange plate thickness is $t_f = 1.75$ in.; and bolt and washer diameters are $d_b = 1.125$ in., $d_w = 1.875$ in.

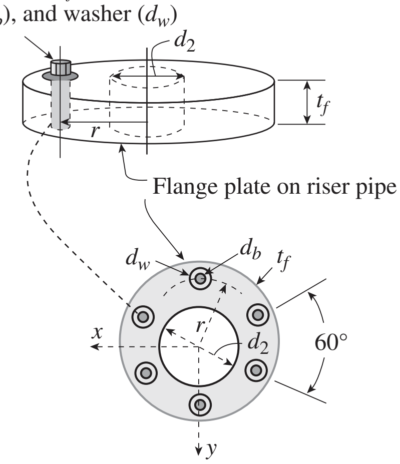
(a) If the entire length of the riser pipe were suspended in air, find the average normal stress σ in each bolt, the average bearing stress σ_b beneath each washer, and the average shear stress τ through the flange plate at each bolt location for the topmost bolted connection.

(b) If the same riser pipe hangs from a drill rig at sea, what are the normal, bearing, and shear stresses in the connection? (Obtain the weight densities of steel and sea water from Table I-1, Appendix I. Neglect the effect of buoyant foam casings on the riser pipe).



Riser pipe (d_2, d_1 L)

(a)



Flange plate on riser pipe–
plan view (n = 6 *bolts shown*)

(b)

Photo Courtesy of Transocean

Solution 1.7-11

(a) Pipe suspended in air

$L = 5000 \text{ ft} \quad \gamma_s = 490 \text{ lb/ft}^3 \qquad \gamma_w = 63.8 \text{ lb/ft}^3$

$d_2 = 16 \text{ in.} \quad d_1 = 15 \text{ in.} \qquad t = \dfrac{d_2 - d_1}{2} = 0.5 \text{ in.} \quad t_f = 1.75 \text{ in.} \qquad A_{pipe} = \dfrac{\pi}{4}(d_2^2 - d_1^2) = 24.347 \text{ in.}^2$

$W_{pipe} = \gamma_s A_{pipe} L = 414.243 \text{ k}$

$n = 6 \qquad d_b = 1.125 \text{ in.} \qquad d_w = 1.875 \text{ in.} \qquad A_b = \dfrac{\pi}{4} d_b^2 = 0.994 \text{ in.}^2 \qquad A_w = \dfrac{\pi}{4}\left(d_w^2 - d_b^2\right) = 1.8 \text{ in.}^2$

$\boxed{\sigma_b = \dfrac{W_{pipe}}{n\,A_b} = 69.5 \text{ ksi}} \qquad \boxed{\sigma_{brg} = \dfrac{W_{pipe}}{n\,A_w} = 39.1 \text{ ksi}} \qquad \boxed{\tau_f = \dfrac{W_{pipe}}{n\,d_w t_f} = 21 \text{ ksi}}$

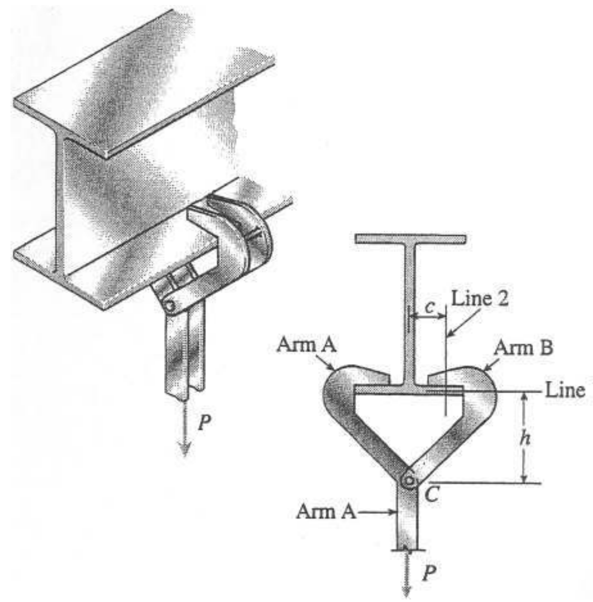
(b) Pipe suspended in sea water $\quad W_{inwater} = (\gamma_s - \gamma_w)\, A_{pipe} L = 360.307 \text{ kip}$

$\boxed{\sigma_b = \dfrac{W_{inwater}}{n A_b} = 60.4 \text{ ksi}} \qquad \boxed{\sigma_{brg} = \dfrac{W_{inwater}}{n A_w} = 34 \text{ ksi}} \qquad \boxed{\tau_f = \dfrac{W_{inwater}}{n\, d_w t_f} = 18.3 \text{ ksi}}$

Problem 1.7-12 The clamp shown in the figure is used to support a load hanging from the lower flange of a steel beam. The clamp consists of two arms (*A* and *B*) joined by a pin at *C*. The pin has diameter $d = 12$ mm. Because arm *B* straddles arm *A*, the pin is in double shear.

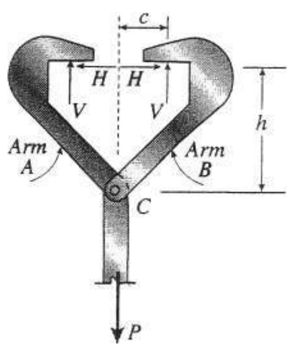
Line 1 in the figure defines the line of action of the resultant horizontal force *H* acting between the lower flange of the beam and arm *B*. The vertical distance from this line to the pin is $h = 250$ mm. Line 2 defines the line of action of the resultant vertical force *V* acting between the flange and arm *B*. The horizontal distance from this line to the centerline of the beam is $c = 100$ mm. The force conditions between arm *A* and the lower flange are symmetrical with those given for arm *B*.

Determine the average shear stress in the pin at *C* when the load $P = 18$ kN.



Solution 1.7-12 Clamp supporting a load *P*

FREE-BODY DIAGRAM OF CLAMP



$h = 250$ mm

$c = 100$ mm

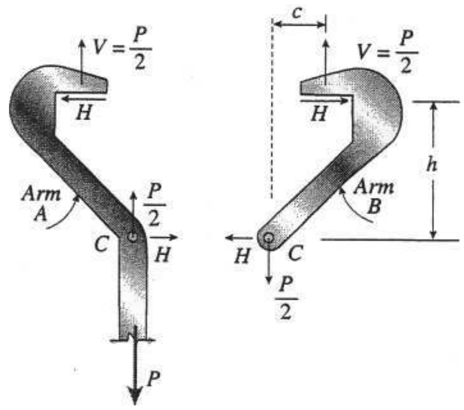
$P = 18$ kN

From vertical equilibrium:

$$V = \frac{P}{2} = 9 \text{ kN}$$

d = diameter of pin at $C = 12$ mm

FREE-BODY DIAGRAMS OF ARMS A AND B

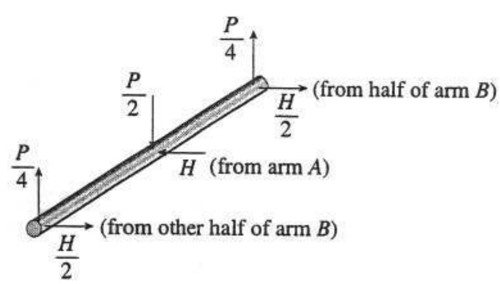


$$\Sigma M_C = 0$$

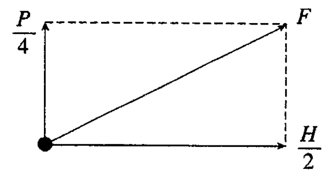
$$V_C - Hh = 0$$

$$H = \frac{V_C}{h} = \frac{P_c}{2h} = 3.6 \text{ kN}$$

FREE-BODY DIAGRAM OF PIN



SHEAR FORCE F IN PIN



$$F = \sqrt{\left(\frac{P}{4}\right)^2 + \left(\frac{H}{2}\right)^2}$$

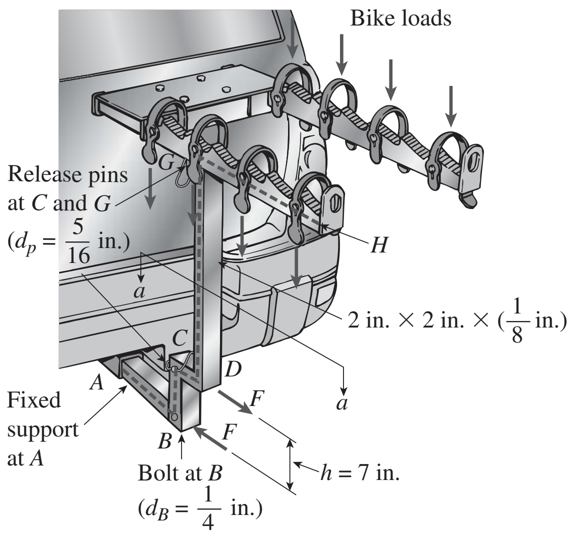
$$= 4.847 \text{ kN}$$

AVERAGE SHEAR STRESS IN THE PIN

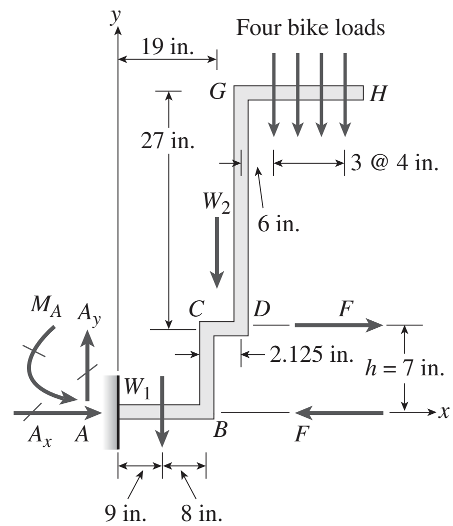
$$\tau_{\text{aver}} = \frac{F}{A_{\text{pin}}} = \frac{F}{\frac{\pi a^2}{4}} = 42.9 \text{ MPa} \quad \leftarrow$$

Problem 1.7-13 A hitch-mounted bicycle rack is designed to carry up to four 30-lb. bikes mounted on and strapped to two arms *GH* [see bike loads in the figure part (a)]. The rack is attached to the vehicle at *A* and is assumed to be like a cantilever beam *ABCDGH* [figure part (b)]. The weight of fixed segment *AB* is $W_1 = 10$ lb, centered 9 in. from *A* [see the figure part (b)] and the rest of the rack weighs $W_2 = 40$ lb, centered 19 in. from *A*. Segment *ABCDG* is a steel tube, 2×2 in., of thickness $t = 1/8$ in. Segment *BCDGH* pivots about a bolt at *B* of diameter $d_B = 0.25$ in. to allow access to the rear of the vehicle without removing the hitch rack. When in use, the rack is secured in an upright position by a pin at *C* (diameter of pin $d_p = 5/16$ in.) [see photo and figure part (c)]. The overturning effect of the bikes on the rack is resisted by a force couple *Fh* at *BC*.

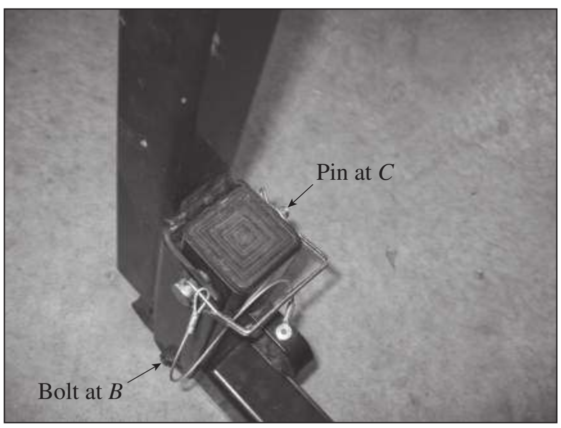
(a) Find the support reactions at *A* for the fully loaded rack.
(b) Find forces in the bolt at *B* and the pin at *C*.
(c) Find average shear stresses τ_{aver} in both the bolt at *B* and the pin at *C*.
(d) Find average bearing stresses σ_b in the bolt at *B* and the pin at *C*.

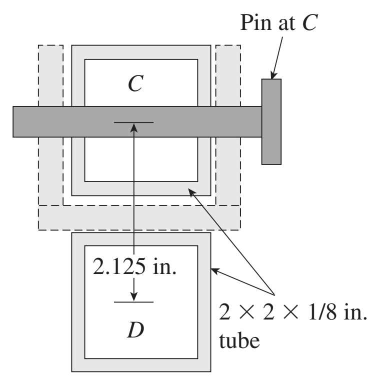


(a)



(b)





(c) Section *a–a*

Solution 1.7-13

NUMERICAL DATA

$t = \frac{1}{8}$ in. $\quad b = 2$ in.

$h = 7$ in. $\quad W_1 = 10$ lb $\quad W_2 = 40$ lb

$P = 30$ lb $\quad d_B = 0.25$ in. $\quad d_p = \frac{5}{16}$ in.

(a) REACTIONS AT *A*

$A_x = 0 \quad \leftarrow$

$A_y = W_1 + W_2 + 4P \quad \leftarrow$

$A_y = 170$ lb $\quad \leftarrow$

$L_1 = 17 + 2.125 + 6 \qquad L_1 = 25$ in.

(dist from *A* to first bike)

$M_A = W_1(9) + W_2(19) + P(4L_1 + 4 + 8 + 12)$

$M_A = 4585$ in.-lb

(b) FORCES IN BOLT AT *B* AND PIN AT *C*

$\Sigma F_y = 0 \qquad B_y = W_2 + 4P \qquad B_y = 160$ lb $\quad \leftarrow$

$\Sigma M_B = 0$

Right hand FBD

$$B_x = \frac{[W_2(19 - 17) + P(6 + 2.125) + P(8.125 + 4) + P(8.125 + 8) + P(8.125 + 12)]}{h}$$

$B_x = 254 \text{ lb} \quad \leftarrow \qquad C_x = -B_x$

$B_{\text{res}} = \sqrt{B_x{}^2 + B_y{}^2} \qquad B_{\text{res}} = 300 \text{ lb} \quad \leftarrow$

(c) Average shear stresses τ_{ave} in both the bolt at B and the pin at C

$A_{sB} = 2\dfrac{\pi d_B^2}{4} \qquad A_{sB} = 0.098 \text{ in.}^2$

$\tau_B = \dfrac{B_{\text{res}}}{A_{sB}} \qquad \tau_B = 3054 \text{ psi} \quad \leftarrow$

$A_{sC} = 2\dfrac{\pi d_p^2}{4} \qquad A_{sC} = 0.153 \text{ in.}^2$

$\tau_C = \dfrac{B_x}{A_{sC}} \qquad \tau_C = 1653 \text{ psi} \quad \leftarrow$

(d) Bearing stresses σ_B in the bolt at B and the pin at C

$t = 0.125$ in.

$A_{bB} = 2td_B \qquad A_{bB} = 0.063 \text{ in.}^2$

$\sigma_{bB} = \dfrac{B_{\text{res}}}{A_{bB}} \qquad \sigma_{bB} = 4797 \text{ psi} \quad \leftarrow$

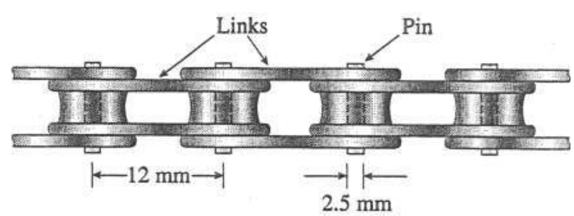
$A_{bC} = 2td_p \qquad A_{bC} = 0.078 \text{ in.}^2$

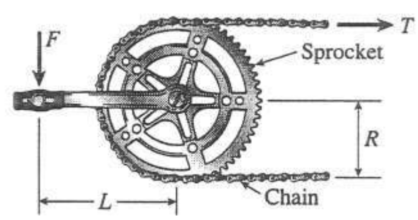
$\sigma_{bC} = \dfrac{C_x}{A_{bC}} \qquad \sigma_{bC} = 3246 \text{ psi} \quad \leftarrow$

Problem 1.7-14 A bicycle chain consists of a series of small links, each 12 mm long between the centers of the pins (see figure). You might wish to examine a bicycle chain and observe its construction. Note particularly the pins, which we will assume to have a diameter of 2.5 mm.

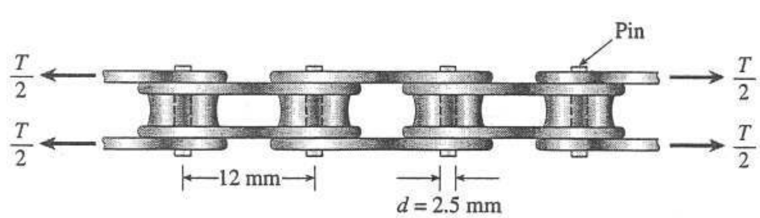
In order to solve this problem, you must now make two measurements on a bicycle (see figure): (1) the length L of the crank arm from main axle to pedal axle, and (2) the radius R of the sprocket (the toothed wheel, sometimes called the chainring).

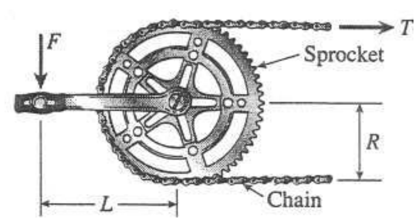
(a) Using your measured dimensions, calculate the tensile force T in the chain due to a force $F = 800$ N applied to one of the pedals.
(b) Calculate the average shear stress τ_{aver} in the pins.





Solution 1.7-14 Bicycle chain





F = force applied to pedal = 800 N

L = length of crank arm

R = radius of sprocket

MEASUREMENTS (FOR AUTHOR'S BICYCLE)

(1) $L = 162$ mm (2) $R = 90$ mm

(a) TENSILE FORCE T IN CHAIN

$$\Sigma M_{\text{axle}} = 0 \quad FL = TR \quad T = \frac{FL}{R}$$

Substitute numerical values:

$$T = \frac{(800\text{ N})(162\text{ mm})}{90\text{ mm}} = 1440\text{ N} \quad \leftarrow$$

(b) SHEAR STRESS IN PINS

$$\tau_{\text{aver}} = \frac{T/2}{A_{\text{pin}}} = \frac{T}{2\dfrac{\pi d^2}{(4)}} = \frac{2T}{\pi d^2}$$

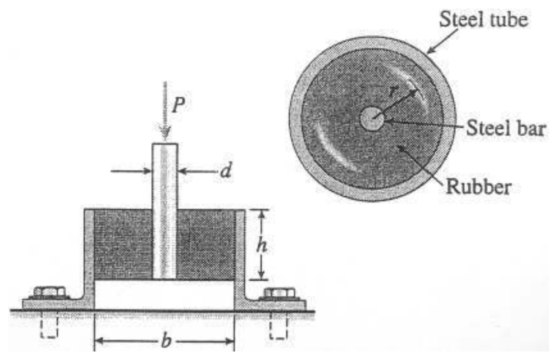
$$= \frac{2FL}{\pi d^2 R}$$

Substitute numerical values:

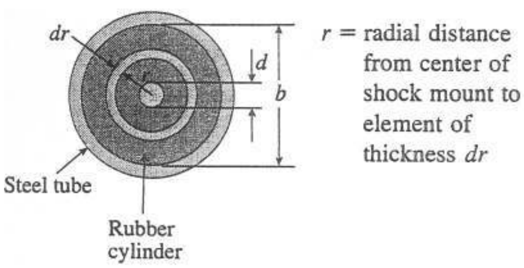
$$\tau_{\text{aver}} = \frac{2(800\text{ N})(162\text{ mm})}{\pi(2.5\text{ mm})^2(90\text{ mm})} = 147\text{ MPa} \quad \leftarrow$$

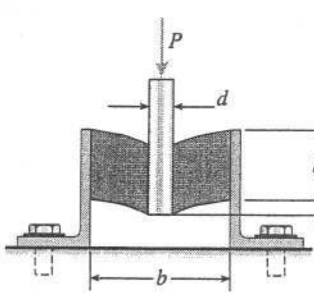
Problem 1.7-15 A shock mount constructed as shown in the figure is used to support a delicate instrument. The mount consists of an outer steel tube with inside diameter b, a central steel bar of diameter d that supports the load P, and a hollow rubber cylinder (height h) bonded to the tube and bar.

(a) Obtain a formula for the shear τ in the rubber at a radial distance r from the center of the shock mount.
(b) Obtain a formula for the downward displacement δ of the central bar due to the load P, assuming that G is the shear modulus of elasticity of the rubber and that the steel tube and bar are rigid.



Solution 1.7-15 Shock mount





r = radial distance from center of shock mount to element of thickness dr

(a) SHEAR STRESS τ AT RADIAL DISTANCE r

A_s = shear area at distance $r = 2\pi rh$

$$\tau = \frac{P}{A_s} = \frac{P}{2\pi rh} \quad \leftarrow$$

(b) DOWNWARD DISPLACEMENT δ

γ = shear strain at distance r

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi rhG}$$

$d\delta$ = downward displacement for element dr

$$d\delta = \gamma dr = \frac{Pdr}{2\pi rhG}$$

$$\delta = \int d\delta = \int_{d/2}^{b/2} \frac{Pdr}{2\pi rhG}$$

$$\delta = \frac{P}{2\pi hG}\int_{d/2}^{b/2}\frac{dr}{r} = \frac{P}{2\pi hG}\,[\ln r]_{d/2}^{b/2}$$

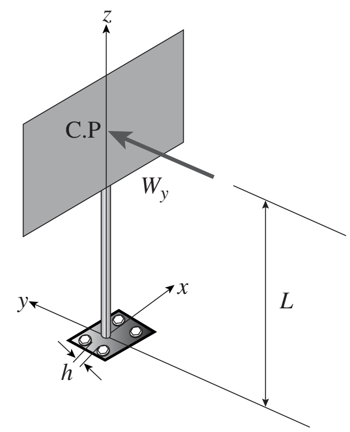
$$\delta = \frac{P}{2\pi hG}\ln\frac{b}{d} \quad \leftarrow$$

Problem 1.7-16 A removable sign post on a hurricane evacuation route has a square base plate with four slots (or cut outs) at bolts 1 through 4 (see drawing and photo) for ease of installation and removal. The upper portion of the post has a separate base plate which is bolted to an anchored base (see photo). Each of the four bolts has diameter d_b and a washer of diameter d_w. The bolts are arranged in a rectangular pattern ($b \times h$). Consider ***wind force*** $\boldsymbol{W_y}$ applied in the y-direction at the center of pressure of the sign structure at a height $z = L$ above the base. Neglect the weight of the sign and post, and also neglect friction between the upper and lower base plates. Assume that the lower base plate and short anchored post are rigid.

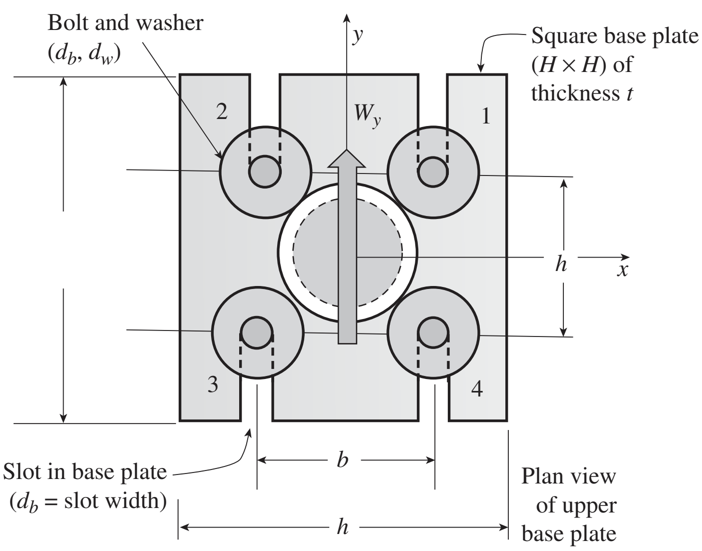
(a) Find average **shear stress** τ, (MPa) at **bolt #1** due to the wind force $\boldsymbol{W_y}$; repeat for **bolt #4**.
(b) Find average **bearing stress** σ_b, (MPa) between the bolt and the base plate (thickness t) at **bolt #1**; repeat for **bolt #4**.
(c) Find average **bearing stress** σ_b, (MPa) between base plate and washer at **bolt #4** due to the wind force $\boldsymbol{W_y}$ (assume initial bolt pretension is zero)
(d) Find average **shear stress** τ, (MPa) through the base plate at **bolt #4** due to the wind force $\boldsymbol{W_y}$.
(e) Find an expression for **normal stress** σ in **bolt #3** due to the wind force $\boldsymbol{W_y}$.

See Prob. 1.8-15 for additional discussion of wind on a sign, and the resulting forces acting on a *conventional* base plate.

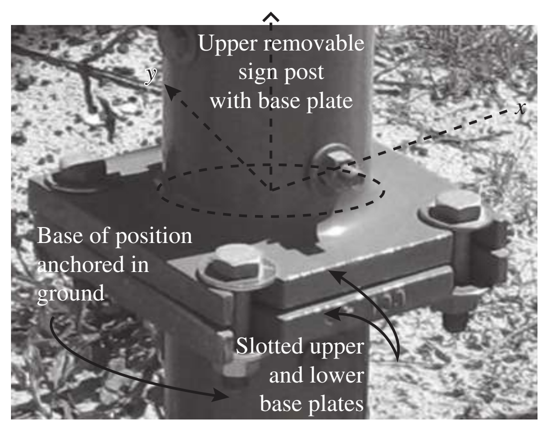
NUMERICAL DATA $L = 2.75$ m $W_y = 667$ N $H = 150$ mm $h = 108$ mm
$b = 96$ mm $t = 14$ mm $d_b = 12$ mm $d_w = 22$ mm



(a)



(b)



Solution 1.7-16

(a) FIND AVERAGE **SHEAR STRESS** (τ, MPa) AT **BOLT #1** DUE TO THE WIND FORCE $\boldsymbol{W_y}$; REPEAT FOR **BOLT #4**

$$A_b = \frac{\pi}{4}d_b^2 = 113.097\text{ mm}^2 \qquad \tau_1 = \frac{\frac{w_y}{2}}{A_b} = 2.95\text{ MPa} \qquad \boxed{\tau_1 = 2.95\text{ MPa}} \qquad \boxed{\tau_4 = 0} \qquad \frac{w_y}{2} = 333.5\text{ N}$$

(only bolts 1 and 2 resist wind force shear in $+y$-direction)

(b) FIND AVERAGE **BEARING STRESS** (σ_b, MPa) BETWEEN THE BOLT AND THE BASE PLATE (THICKNESS t) AT **BOLT #1**; REPEAT FOR **BOLT #4**

$$A_{\text{brg}} = d_b t = 168\text{ mm}^2 \qquad \sigma_{b1} = \frac{\frac{w_y}{2}}{A_{\text{brg}}} = 1.985\text{ MPa} \qquad \boxed{\sigma_{b1} = 1.985\text{ MPa}} \qquad \boxed{\sigma_{b4} = 0}$$

(only bolts 1 and 2 resist wind force bearing in $+y$-direction)

(c) FIND AVERAGE **BEARING STRESS** (σ_b, MPa) BETWEEN BASE PLATE AND WASHER AT **BOLT #4** DUE TO THE WIND FORCE $\boldsymbol{W_y}$ (ASSUME INITIAL BOLT PRETENSION IS ZERO)

Assume wind force creates overturning moment about x axis = OTM_x $\qquad \text{OTM}_x = W_y L = 1834.25\text{ N·m}$

OTM is resisted by force couples pairs at bolts 1 to 4 and 2 to 3; so force in bolt 4 is

$$F_4 = \frac{\text{OTM}_x}{2h} = 8491.898\text{ N}$$

Bearing area is donut shaped area of washer in contact with the plate minus approximate rectangular cutout for slot

$$A_{\text{brg}} = \frac{\pi}{4}\left(d_w^{\,2} - d_b^{\,2}\right) - d_b\left(\frac{d_w - d_b}{2}\right) = 207.035\text{ mm}^2 \qquad \sigma_{b4} = \frac{F_4}{A_{\text{brg}}} = 41\text{ MPa} \qquad \boxed{\sigma_{b4} = 41\text{ MPa}}$$

(d) FIND AVERAGE **SHEAR STRESS** (τ, MPa) THROUGH THE BASE PLATE AT **BOLT #4** DUE TO THE WIND FORCE $\boldsymbol{W_y}$;

Use force F_4 above; shear stress is on cylindrical surface at perimeter of washer; must deduct approximate rectangular area due to slot

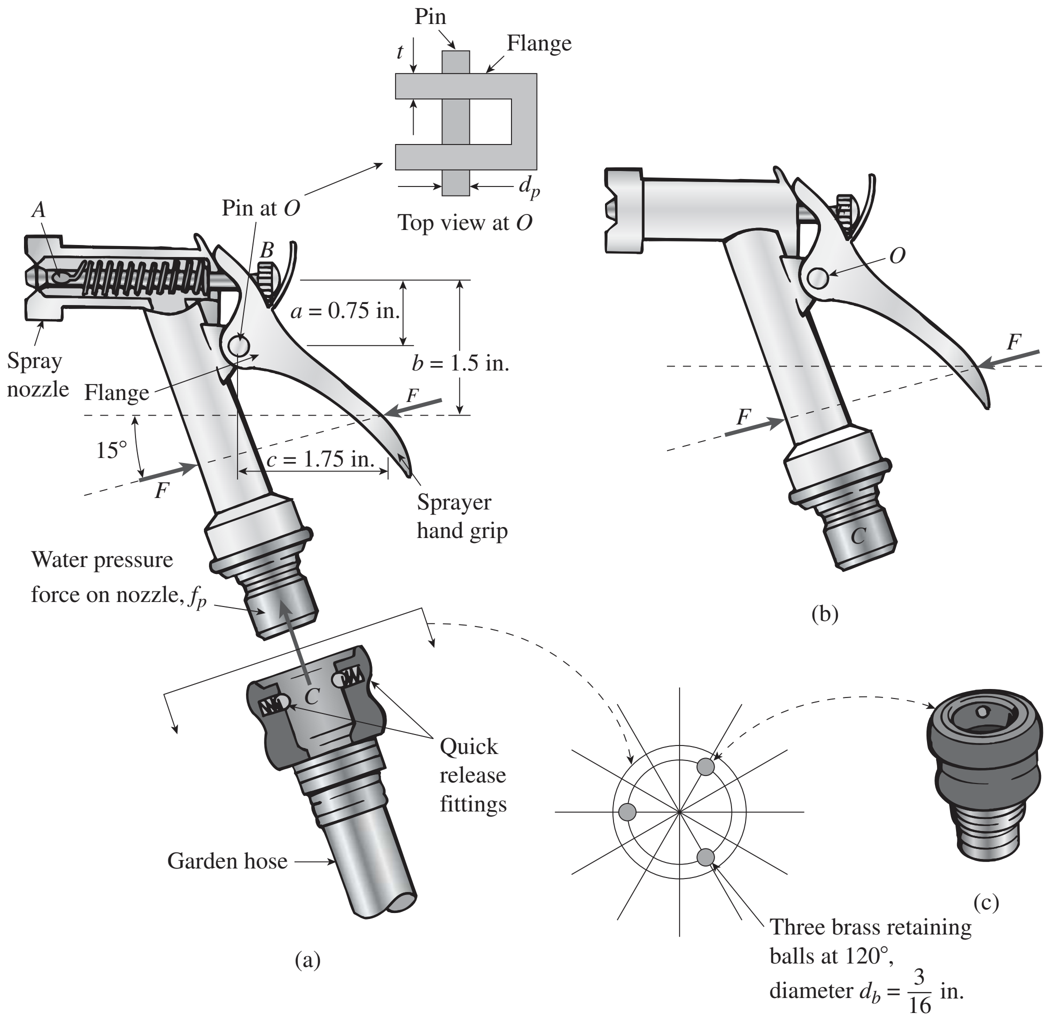
$$A_{\text{sh}} = \left(\pi d_w - d_b\right)t = 799.611\text{ mm}^2 \qquad \tau = \frac{F_4}{A_{\text{sh}}} = 10.62\text{ MPa} \qquad \boxed{\tau = 10.62\text{ MPa}}$$

(e) FIND AN EXPRESSION FOR **NORMAL STRESS** (σ) IN **BOLT #3** DUE TO THE WIND FORCE $\boldsymbol{W_y}$.

Force in bolt #3 due to OTM_x is same as that in bolt #4 $\qquad \sigma_3 = \dfrac{F_4}{A_b} = 75.1\text{ MPa} \qquad \boxed{\sigma_3 = 75.1\text{ MPa}}$

Problem 1.7-17 A spray nozzle for a garden hose requires a force $F = 5$ lb. to open the spring-loaded spray chamber AB. The nozzle hand grip pivots about a pin through a flange at O. Each of the two flanges has thickness $t = 1/16$ in., and the pin has diameter $d_p = 1/8$ in. [see figure part (a)]. The spray nozzle is attached to the garden hose with a quick release fitting at B [see figure part (b)]. Three brass balls (diameter $d_b = 3/16$ in.) hold the spray head in place under water pressure force $f_p = 30$ lb at C [see figure part (c)]. Use dimensions given in figure part (a).

(a) Find the force in the pin at O due to applied force F.
(b) Find average shear stress τ_{aver} and bearing stress σ_b in the pin at O.



Solution 1.7-17

NUMERICAL DATA

$F = 5 \text{ lb} \quad t = \frac{1}{16} \text{ in.} \quad d_p = \frac{1}{8} \text{ in.} \quad d_b = \frac{3}{16} \text{ in.}$

$f_p = 30 \text{ lb} \quad d_N = \frac{5}{8} \text{ in.} \quad \theta = 15\frac{\pi}{180} \text{ rad}$

$a = 0.75 \text{ in.} \quad b = 1.5 \text{ in.} \quad c = 1.75 \text{ in.}$

(a) FIND THE FORCE IN THE PIN AT O DUE TO APPLIED FORCE F

$\sum M_o = 0$

$$F_{AB} = \frac{[F\cos(\theta)(b - a)] + F\sin(\theta)(c)}{a}$$

$F_{AB} = 7.849 \text{ lb}$

$\sum F_H = 0 \qquad O_x = F_{AB} + F\cos(\theta)$

$O_y = F\sin(\theta)$

$O_x = 12.68 \text{ lb} \quad O_y = 1.294 \text{ lb}$

$O_{res} = \sqrt{O_x^2 + O_y^2} \qquad O_{res} = 12.74 \text{ lb} \quad \leftarrow$

(b) FIND AVERAGE SHEAR STRESS τ_{ave} AND BEARING STRESS σ_b IN THE PIN AT O

$A_s = 2\frac{\pi d_p^2}{4} \quad \tau_O = \frac{O_{res}}{A_s} \quad \tau_O = 519 \text{ psi} \quad \leftarrow$

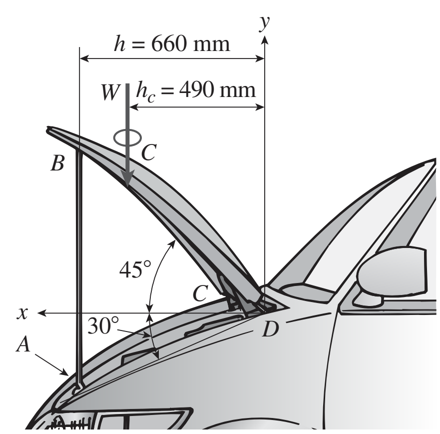
$A_b = 2td_p \quad \sigma_{bO} = \frac{O_{res}}{A_b} \quad \sigma_{bO} = 816 \text{ psi} \quad \leftarrow$

(c) FIND THE AVERAGE SHEAR STRESS τ_{ave} IN THE BRASS RETAINING BALLS AT B DUE TO WATER PRESSURE FORCE F_p

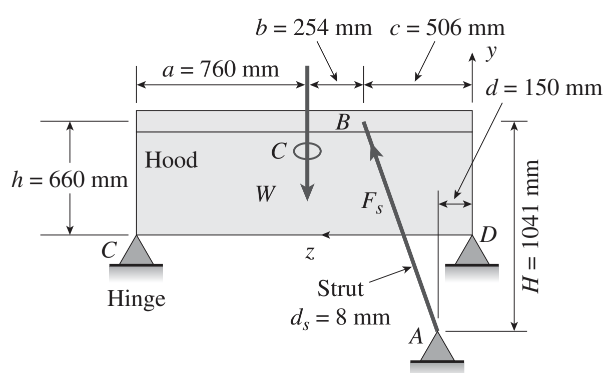
$A_s = 3\frac{\pi d_b^2}{4} \quad \tau_{ave} = \frac{f_p}{A_s} \quad \tau_{ave} = 362 \text{ psi} \quad \leftarrow$

Problem 1.7-18 A single steel strut AB with diameter $d_s = 8$ mm. supports the vehicle engine hood of mass 20 kg which pivots about hinges at C and D [see figures (a) and (b)]. The strut is bent into a loop at its end and then attached to a bolt at A with diameter $d_b = 10$ mm. Strut AB lies in a vertical plane.

(a) Find the strut force F_s and average normal stress σ in the strut.
(b) Find the average shear stress τ_{aver} in the bolt at A.
(c) Find the average bearing stress σ_b on the bolt at A.



(a)



(b)

Solution 1.7-18

NUMERICAL DATA

$d_s = 8$ mm $\quad d_b = 10$ mm $\quad m = 20$ kg

$a = 760$ mm $\quad b = 254$ mm

$c = 506$ mm $\quad d = 150$ mm

$h = 660$ mm $\quad h_c = 490$ mm

$$H = h\left(\tan\left(30\frac{\pi}{180}\right) + \tan\left(45\frac{\pi}{180}\right)\right)$$

$H = 1041$ mm

$W = m\,(9.81\text{m/s}^2) \quad W = 196.2$ N

$$\frac{a + b + c}{2} = 760 \text{ mm}$$

VECTOR r_{AB}

$$r_{AB} = \begin{pmatrix} 0 \\ \text{H} \\ \text{c} - \text{d} \end{pmatrix} \qquad r_{AB} = \begin{pmatrix} 0 \\ 1.041 \times 10^3 \\ 356 \end{pmatrix}$$

UNIT VECTOR e_{AB}

$$e_{AB} = \frac{r_{AB}}{|r_{AB}|} \qquad e_{AB} = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix} \qquad |e_{AB}| = 1$$

$$W = \begin{pmatrix} 0 \\ -W \\ 0 \end{pmatrix} \qquad W = \begin{pmatrix} 0 \\ -196.2 \\ 0 \end{pmatrix}$$

$$r_{DC} = \begin{pmatrix} h_c \\ h_c \\ b + c \end{pmatrix} \qquad r_{DC} = \begin{pmatrix} 490 \\ 490 \\ 760 \end{pmatrix}$$

$$\sum M_D \qquad M_D = r_{DB} \times F_s e_{AB} + W \times r_{DC}$$

(ignore force at hinge C since it will vanish with moment about line DC)

$$F_{sx} = 0 \qquad F_{sy} = \frac{H}{\sqrt{H^2 + (c - d)^2}} F_s$$

$$F_{sz} = \frac{c - d}{\sqrt{H^2 + (c - d)^2}} F_s$$

where

$$\frac{H}{\sqrt{H^2 + (c - d)^2}} = 0.946$$

$$\frac{c - d}{\sqrt{H^2 + (c - d)^2}} = 0.324$$

(a) FIND THE STRUT FORCE F_S AND AVERAGE NORMAL STRESS σ IN THE STRUT

$$\sum M_{\text{line}DC} = 0 \qquad F_{sy} = \frac{|W|h_c}{h}$$

$F_{sy} = 145.664$

$$F_s = \frac{F_{sy}}{\dfrac{H}{\sqrt{H^2 + (c - d)^2}}} \qquad F_s = 153.9 \text{ N} \quad \longleftarrow$$

$$A_{\text{strut}} = \frac{\pi}{4} d_s^2 \qquad A_{\text{strut}} = 50.265 \text{ mm}^2$$

$$\sigma = \frac{F_s}{A_{\text{strut}}} \qquad \sigma = 3.06 \text{ MPa} \quad \longleftarrow$$

(b) FIND THE AVERAGE SHEAR STRESS τ_{ave} IN THE BOLT AT A

$d_b = 10$ mm

$$A_s = \frac{\pi}{4} d_b^2 \qquad A_s = 78.54 \text{ mm}^2$$

$$\tau_{\text{ave}} = \frac{F_s}{A_s} \qquad \tau_{\text{ave}} = 1.96 \text{ MPa} \quad \longleftarrow$$

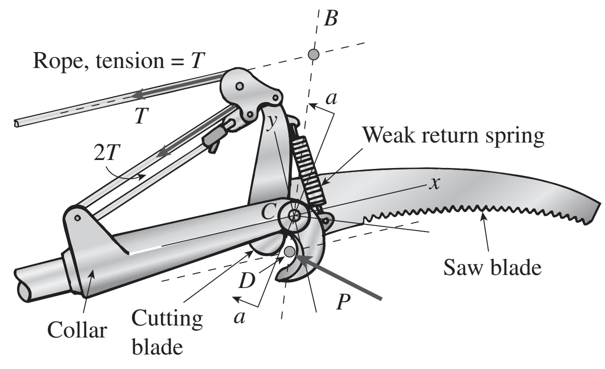
(c) FIND THE BEARING STRESS σ_b ON THE BOLT AT A

$A_b = d_s d_b \qquad A_b = 80 \text{ mm}^2$

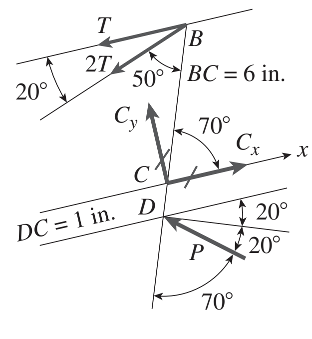
$$\sigma_b = \frac{F_s}{A_b} \qquad \sigma_b = 1.924 \text{ MPa} \quad \longleftarrow$$

Problem 1.7-19 The top portion of a pole saw used to trim small branches from trees is shown in the figure part (a). The cutting blade *BCD* [see figure parts (a) and (c)] applies a force *P* at point *D*. Ignore the effect of the weak return spring attached to the cutting blade below *B*. Use properties and dimensions given in the figure.

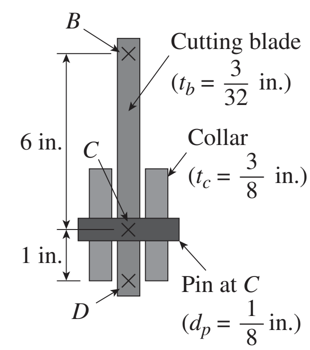
(a) Find the force *P* on the cutting blade at *D* if the tension force in the rope is $T = 25$ lb (see free body diagram in part (b)].
(b) Find force in the pin at *C*.
(c) Find average shear stress τ_{ave} and bearing stress σ_b in the support pin at *C* [see Section *a–a* through cutting blade in figure part (c)].



(a) Top part of pole saw



(b) Free-body diagram



(c) Section *a–a*

Solution 1.7-19

Numerical properties

$d_p = \dfrac{1}{8}$ in. $\quad t_b = \dfrac{3}{32}$ in. $\quad t_c = \dfrac{3}{8}$ in.

$T = 25$ lb $\quad d_{BC} = 6$ in.

$d_{CD} = 1$ in.

(a) Find the cutting force *P* on the cutting blade at *D* if the tension force in the rope is $T = 25$ lb:

$$\sum M_c = 0$$

$$M_C = T(6\sin(70°)) + 2T\cos(20°)(6\sin(70°)) - 2T\sin(20°)(6\cos(70°)) - P\cos(20°)(1)$$

Solve equation for *P*

$$P = \frac{[T(6\sin(70°)) + 2T\cos(20°)6\sin(70°)) - 2T\sin(20°)(6\cos(70°))]}{\cos(20°)}$$

$$P = 395 \text{ lbs} \quad \leftarrow$$

(b) Solve for forces on pin at *C*

$$\sum F_x = 0 \quad C_x = T + 2T\cos(20°) + P\cos(40°)$$

$$C_x = 374 \text{ lbs} \quad \leftarrow$$

$$\sum F_y = 0 \quad C_y = 2T\sin(20°) - P\sin(40°)$$

$$C_y = -237 \text{ lbs} \quad \leftarrow$$

RESULTANT AT C

$$C_{res} = \sqrt{C_x^2 + C_y^2} \qquad C_{res} = 443 \text{ lbs} \quad \leftarrow$$

(c) FIND MAXIMUM SHEAR AND BEARING STRESSES IN THE SUPPORT PIN AT C (SEE SECTION A–A THROUGH SAW).

SHEAR STRESS—PIN IN DOUBLE SHEAR

$$A_s = \frac{\pi}{4} d_p^2 \qquad A_s = 0.012 \text{ in.}^2$$

$$\tau_{ave} = \frac{C_{res}}{2A_s} \qquad \tau_{ave} = 18.04 \text{ ksi}$$

BEARING STRESSES ON PIN ON EACH SIDE OF COLLAR

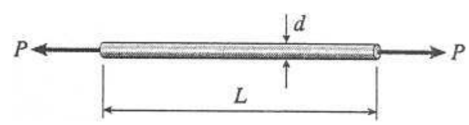
$$\sigma_{bC} = \frac{\frac{C_{res}}{2}}{d_p t_c} \qquad \sigma_{bC} = 4.72 \text{ ksi} \quad \leftarrow$$

BEARING STRESS ON PIN AT CUTTING BLADE

$$\sigma_{bcb} = \frac{C_{res}}{d_p t_b} \qquad \sigma_{bcb} = 37.8 \text{ ksi} \quad \leftarrow$$

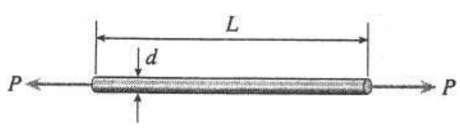
Allowable Stresses and Allowable Loads

Problem 1.8-1 A bar of solid circular cross section is loaded in tension by forces P (see figure). The bar has length $L = 16.0$ in. and diameter $d = 0.50$ in. The material is a magnesium alloy having modulus of elasticity $E = 6.4 \times 10^6$ psi. The allowable stress in tension is $\sigma_{allow} = 17{,}000$ psi, and the elongation of the bar must not exceed 0.04 in.



What is the allowable value of the forces P?

Solution 1.8-1 Magnesium bar in tension



$L = 16.0$ in. $\qquad d = 0.50$ in.

$E = 6.4 \times 10^6$ psi

$\sigma_{allow} = 17{,}000$ psi $\qquad \delta_{max} = 0.04$ in.

MAXIMUM LOAD BASED UPON ELONGATION

$$\varepsilon_{max} = \frac{\delta_{max}}{L} = \frac{0.04 \text{ in.}}{16 \text{ in.}} 0.00250$$

$$\sigma_{max} = E\epsilon_{max} = (6.4 \times 10^6 \text{ psi})(0.00250)$$

$$= 16{,}000 \text{ psi}$$

$$P_{max} = \sigma_{max} A = (16.000 \text{ psi})\left(\frac{\pi}{4}\right)(0.50 \text{ in.})^2$$

$$= 3140 \text{ lb}$$

MAXIMUM LOAD BASED UPON TENSILE STRESS

$$P_{max} = \sigma_{allow} A = (17{,}000 \text{ psi})\left(\frac{\pi}{4}\right)(0.50 \text{ in.})^2$$

$$= 3340 \text{ Ib}$$

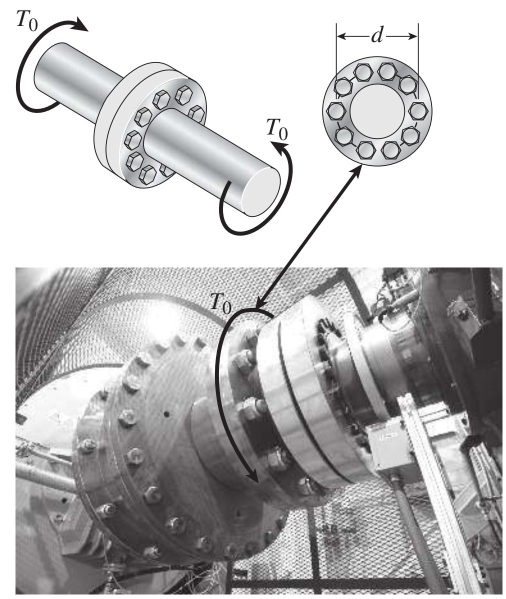
ALLOWABLE LOAD

Elongation governs.

$P_{allow} = 3140$ lb $\quad \leftarrow$

Problem 1.8-2 A torque T_0 is transmitted between two flanged shafts by means of ten 20-mm bolts (see figure and photo). The diameter of the bolt circle is $d = 250$ mm.

If the allowable shear stress in the bolts is 90 MPa, what is the maximum permissible torque? (Disregard friction between the flanges.)



Solution 1.8-2 Shafts with flanges

NUMERICAL DATA

$r = 10 \quad d = 250$ mm

^ bolts ^ flange

$A_s = \pi r^2$

$A_s = 314.159\ \mathrm{m}^2$

$\tau_a = 85$ MPa

MAXIMUM PERMISSIBLE TORQUE

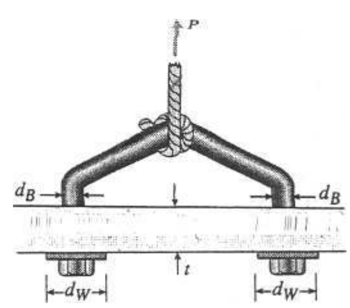
$$T_{max} = \tau_a A_s \left(r \frac{d}{2} \right)$$

$$T_{max} = 3.338 \times 10^7\ \mathrm{N{\cdot}mm}$$

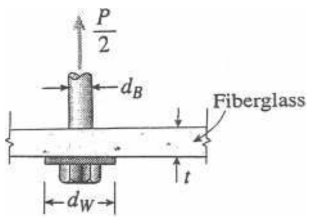
$$T_{max} = 33.4\ \mathrm{kN{\cdot}m} \quad \leftarrow$$

Problem 1.8-3 A tie-down on the deck of a sailboat consists of a bent bar bolted at both ends, as shown in the figure. The diameter d_B of the bar is $^1/_4$ in., the diameter d_W of the washers is $^7/_8$ in., and the thickness t of the fiberglass deck is $^3/_8$ in.

If the allowable shear stress in the fiberglass is 300 psi, and the allowable bearing pressure between the washer and the fiberglass is 550 psi, what is the allowable load P_{allow} on the tie-down?



Solution 1.8-3 Bolts through fiberglass



$$d_B = \frac{1}{4}\text{ in.}$$

$$d_W = \frac{7}{8}\text{ in.}$$

$$t = \frac{3}{8}\text{ in.}$$

Allowable load based upon shear stress in fiberglass

$\tau_{allow} = 300$ psi

Shear area $A_s = \pi d_W t$

$$\frac{P_1}{2} = \tau_{allow} A_s = \tau_{allow}(\pi d_W t)$$

$$= (300\text{ psi})(\pi)\left(\frac{7}{8}\text{ in.}\right)\left(\frac{3}{8}\text{ in.}\right)$$

$$\frac{P_1}{2} = 309.3\text{ lb}$$

$P_1 = 619$ lb

Allowable load based upon bearing pressure

$\sigma_b = 550$ psi

Bearing area $A_b = \frac{\pi}{4}(d_W^2 - d_B^2)$

$$\frac{P_2}{2} = \sigma_b A_b = (550\text{ psi})\left(\frac{\pi}{4}\right)\left[\left(\frac{7}{8}\text{ in.}\right)^2 - \left(\frac{1}{4}\text{ in.}\right)^2\right]$$

$$= 303.7\text{ lb}$$

$P_2 = 607$ lb

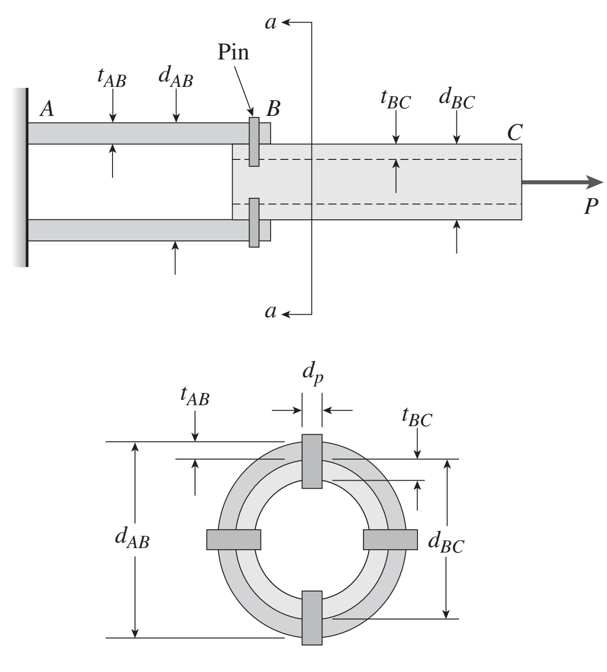
Allowable load

Bearing pressure governs.

$P_{allow} = 607$ lb ←

Problem 1.8-4 Two steel tubes are joined at B by four pins ($d_p = 11$ mm), as shown in the cross section a–a in the figure. The outer diameters of the tubes are $d_{AB} = 41$ mm and $d_{BC} = 28$ mm. The wall thicknesses are $t_{AB} = 6.5$ mm and $t_{BC} = 7.5$ mm. The yield stress in tension for the steel is $\sigma_Y = 200$ MPa and the ultimate stress in *tension* is $\sigma_U = 340$ MPa. The corresponding yield and ultimate values in *shear* for the pin are 80 MPa and 140 MPa, respectively. Finally, the yield and ultimate values in *bearing* between the pins and the tubes are 260 MPa and 450 MPa, respectively. Assume that the factors of safety with respect to yield stress and ultimate stress are 3.5 and 4.5, respectively.

(a) Calculate the allowable tensile force P_{allow} considering tension in the tubes.
(b Recompute P_{allow} for shear in the pins.
(c) Finally, recompute P_{allow} for bearing between the pins and the tubes. Which is the controlling value of P?



Section a–a

Solution 1.8-4

Yield and ultimate stresses (all in MPa) TUBES AND PIN DIMENSIONS (MM)

TUBES: $\sigma_Y = 200$ $\sigma_u = 340$ $FS_y = 3.5$ $d_{AB} = 41$ $t_{AB} = 6.5$

PIN (SHEAR): $\tau_Y = 8$ $\tau_u = 140$ $FS_u = 4.5$ $d_{BC} = d_{AB} - 2\,t_{AB}$ $d_{BC} = 28$

PIN (BEARING): $\sigma_{bY} = 260$ $\sigma_{bu} = 450$ $t_{BC} = 7.5$ $d_p = 11$

(a) P_{ALLOW} CONSIDERING TENSION IN THE TUBES

$$A_{\text{net}AB} = \frac{\pi}{4}\left[d_{AB}^{\,2} - (d_{AB} - 2\,t_{AB})^2\right] - 4\,d_p t_{AB} \qquad A_{\text{net}AB} = 418.502\ \text{mm}^2$$

$$A_{\text{net}BC} = \frac{\pi}{4}\left[d_{BC}^{\,2} - (d_{BC} - 2\,t_{BC})^2\right] - 4\,d_p t_{BC} \qquad A_{\text{net}BC} = 153.02 \quad < \text{ use smaller}$$

$$P_{aT1} = \frac{\sigma_y}{FS_y} A_{\text{net}BC} \qquad P_{aT1} = 8743.993\ \text{N} \quad < \text{controls} \qquad \boxed{P_{\text{allow}} = 8.74\ \text{kN}}$$

$$P_{aT2} = \frac{\sigma_u}{FS_u} A_{\text{net}BC} \qquad P_{aT2} = 11{,}561.501\ \text{N}$$

(b) P_{allow} CONSIDERING SHEAR IN THE PINS $A_s = \frac{\pi}{4} d_p^2$ $A_s = 95.033\ \text{mm}^2$

$$P_{aS1} = (4A_s)\frac{\tau_Y}{FS_y} \qquad P_{aS1} = 8688.748\ \text{N} \quad < \text{controls} \qquad \boxed{P_{\text{allow}} = 8.69\ \text{kN}}$$

$$P_{as2} = (4A_s)\frac{\tau_u}{FS_u} \qquad P_{as2} = 11{,}826.351\ \text{N}$$

(c) P_{allow} CONSIDERING BEARING IN THE PINS

$$A_{bAB} = 4\,d_p\,t_{AB} \qquad A_{bAB} = 286\ \text{mm}^2 \quad < \text{smaller controls}$$

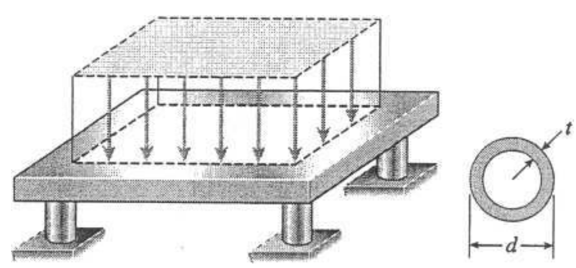
$$A_{bBC} = 4\,d_p\,t_{BC} \qquad A_{bBC} = 330$$

$$P_{ab1} = A_{bAB}\left(\frac{\sigma_{by}}{FS_y}\right) \qquad P_{ab1} = 21{,}245.714\ \text{N} \quad < \text{controls} \qquad \boxed{P_{\text{allow}} = 21.2\ \text{kN}}$$

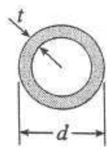
$$P_{ab2} = A_{bAB}\left(\frac{\sigma_{bu}}{FS_u}\right) \qquad P_{ab2} = 28{,}600\ \text{N} \qquad \boxed{\text{Overall, shear controls (Part (b))}}$$

Problem 1.8-5 A steel pad supporting heavy machinery rests on four short, hollow, cast iron piers (see figure). The ultimate strength of the cast iron in compression is 50 ksi. The outer diameter of the piers is $d = 4.5$ in. and the wall thickness is $t = 0.40$ in.

Using a factor of safety of 3.5 with respect to the ultimate strength, determine the total load P that may be supported by the pad.



Solution 1.8-5 Cast iron piers in compression



Four piers

$\sigma_U = 50$ ksi

$n = 3.5$

$$\sigma_{allow} = \frac{\sigma_U}{n} = \frac{50 \text{ ksi}}{3.5} = 14.29 \text{ ksi}$$

$d = 4.5$ in.

$t = 0.4$ in.

$d_0 = d - 2t = 3.7$ in.

$$A = \frac{\pi}{4}(d^2 - d_0^2) = \frac{\pi}{4}[(4.5 \text{ in.})^2 - (3.7 \text{ in.})^2]$$

$$= 5.152 \text{ in.}^2$$

P_1 = allowable load on one pier

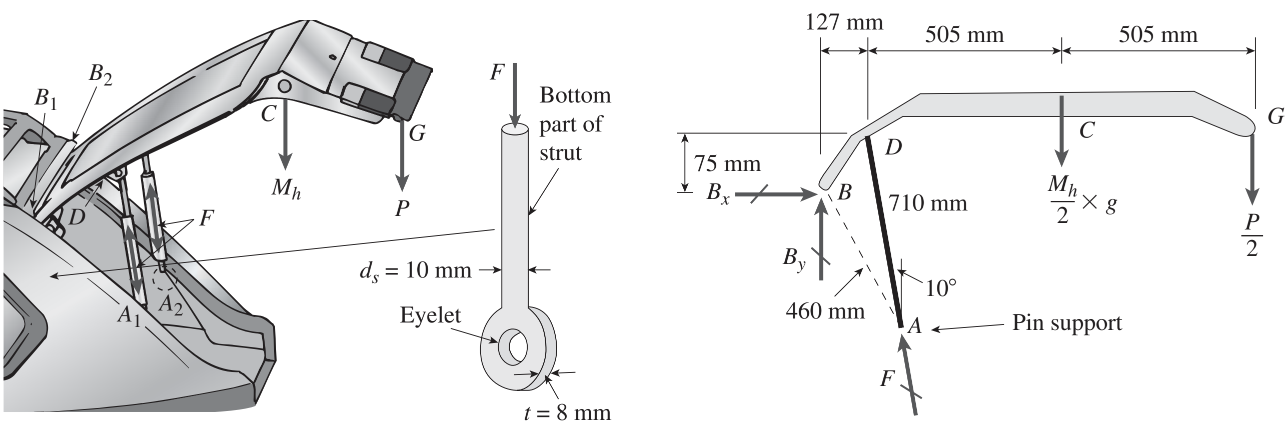
$$= \sigma_{allow} A = (14.29 \text{ ksi})(5.152 \text{ in.}^2)$$

$$= 73.62 \text{ k}$$

Total load $P = 4P_1 = 294$ k $\leftarrow$

Problem 1.8-6 The rear hatch of a van [$BDCF$ in figure part (a)] is supported by two hinges at B_1 and B_2 and by two struts A_1B_1 and A_2B_2 (diameter $d_s = 10$ mm) as shown in figure part (b). The struts are supported at A_1 and A_2 by pins, each with diameter $d_p = 9$ mm and passing through an eyelet of thickness $t = 8$ mm at the end of the strut [figure part (b)]. If a closing force $P = 50$ N is applied at G and the mass of the hatch $M_h = 43$ kg is concentrated at C:

(a) What is the force F in each strut? [Use the free-body diagram of one half of the hatch in the figure part (c)]
(b) What is the maximum permissible force in the strut, F_{allow}, if the allowable stresses are as follows: compressive stress in the strut, 70 MPa; shear stress in the pin, 45 MPa; and bearing stress between the pin and the end of the strut, 110 MPa.



(a) (b) (c)

Solution 1.8-6

Numerical data

$M_h = 43$ kg $\quad \sigma_a = 70$ MPa

$\tau_a = 45$ MPa $\quad \sigma_{ba} = 110$ MPa

$d_s = 10$ mm $\quad d_p = 9$ mm $\quad t = 8$ mm

$P = 50$ N $\quad g = 9.81$ m/s^2

(a) Force F in each strut from statics (sum moments about B)

$$F_V = F\cos(10°) \quad F_H = F\sin(10°)$$

$$\sum M_B = 0$$

$$F_V(127) + F_H(75) = \frac{M_h}{2} g\,(127 + 505) + \frac{P}{2}[127 + 2(505)]$$

$$F(127\cos(10°) + 75\sin(10°))$$

$$= \frac{M_h}{2} g\,(127 + 505) + \frac{P}{2} + 2(505)]$$

$$F = \frac{\frac{M_h}{2} g\,(127 + 505) + \frac{P}{2}[127 + 2(505)]}{(127\cos(10°) + 75\sin(10°))}$$

$$F = 1.171 \text{ kN} \quad \leftarrow$$

(b) Maximum permissible force F in each strut
F_{max} is smallest of the following

$$F_{a1} = \sigma_a \frac{\pi}{4} d_s^2 \quad F_{a1} = 5.50 \text{ kN}$$

$$F_{a2} = \tau_a \frac{\pi}{4} d_p^2$$

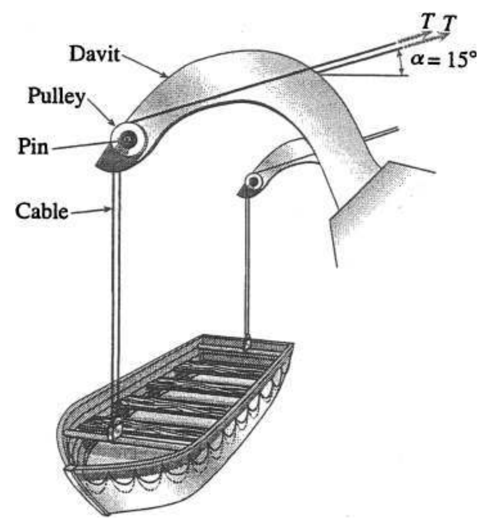
$$F_{a2} = 2.86 \text{ kN} \quad \leftarrow \quad \frac{F_{a2}}{F} = 2.445$$

$$F_{a3} = \sigma_{ba} d_p t \quad F_{a3} = 7.92 \text{ kN}$$

Problem 1.8-7 A lifeboat hangs from two ship's davits, as shown in the figure. A pin of diameter $d = 0.80$ in. passes through each davit and supports two pulleys, one on each side of the davit.

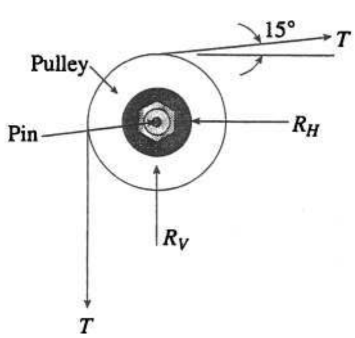
Cables attached to the lifeboat pass over the pulleys and wind around winches that raise and lower the lifeboat. The lower parts of the cables are vertical and the upper parts make an angle $\alpha = 15°$ with the horizontal. The allowable tensile force in each cable is 1800 lb, and the allowable shear stress in the pins is 4000 psi.

If the lifeboat weighs 1500 lb, what is the maximum weight that should be carried in the lifeboat?



Solution 1.8-7 Lifeboat supported by four cables

Free-body diagram of one pulley



Pin diameter $d = 0.80$ in.

T = tensile force in one cable

$T_{allow} = 1800$ lb

$\tau_{allow} = 4000$ psi

W = weight of lifeboat

$= 1500$ lb

$\Sigma F_{horiz} = 0 \qquad R_H = T\cos 15° = 0.9659T$

$\Sigma F_{vert} = 0 \qquad R_V = T - T\sin 15° = 0.7412T$

V = shear force in pin

$V = \sqrt{(R_H)^2 + (R_v)^2} = 1.2175T$

ALLOWABLE TENSILE FORCE IN ONE CABLE BASED UPON SHEAR IN THE PINS

$$V_{allow} = \tau_{allow}\,A_{pin} = (4000\text{ psi})\left(\frac{\pi}{4}\right)(0.80\text{ in.})^2$$

$$= 2011\text{ lb}$$

$$V = 1.2175T \qquad T_1 = \frac{V_{allow}}{1.2175} = 1652\text{ lb}$$

ALLOWABLE FORCE IN ONE CABLE BASED UPON TENSION IN THE CABLE

$$T_2 = T_{allow} = 1800\text{ lb}$$

MAXIMUM WEIGHT

Shear in the pins governs.

$$T_{max} = T_1 = 1652\text{ lb}$$

Total tensile force in four cables

$$= 4T_{max} = 6608\text{ lb}$$

$$W_{max} = 4T_{max} - W$$

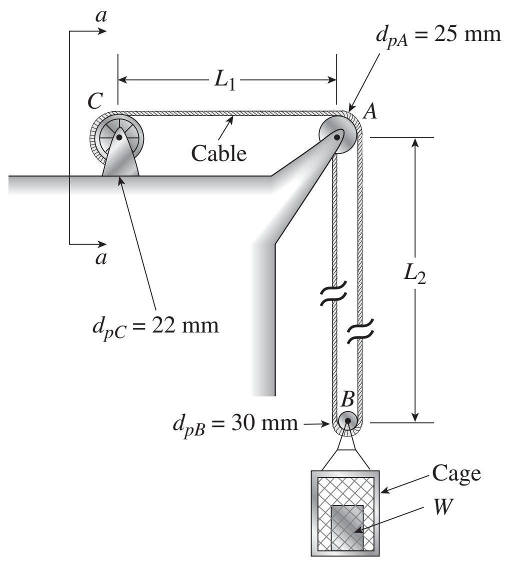
$$= 6608\text{ lb} - 1500\text{ lb}$$

$$= 5110\text{ lb} \quad \leftarrow$$

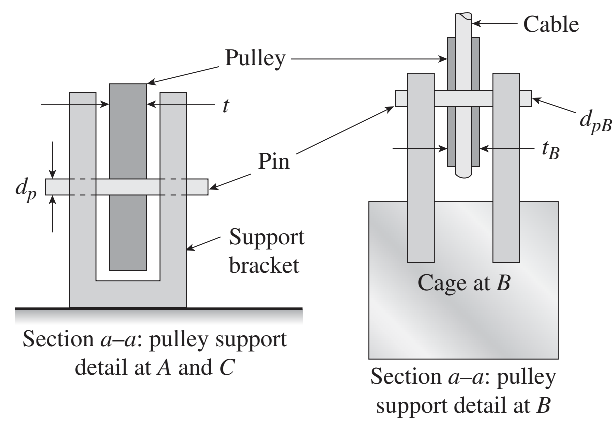
Problem 1.8-8 A cable and pulley system in figure part (a) supports a cage of mass 300 kg at B. Assume that this includes the mass of the cables as well. The thickness of each the three steel pulleys is $t = 40$ mm. The pin diameters are $d_{pA} = 25$ mm, $d_{pB} = 30$ mm and $d_{pC} = 22$ mm [see figure, parts (a) and part (b)].

(a) Find expressions for the resultant forces acting on the pulleys at A, B, and C in terms of cable tension T.

(b) What is the maximum weight W that can be added to the cage at B based on the following allowable stresses? Shear stress in the pins is 50 MPa; bearing stress between the pin and the pulley is 110 MPa.



(a)



(b)

Solution 1.8-8

NUMERICAL DATA

$M = 300\text{ kg}$ $\quad g = 9.81\text{ m/s}^2$

$\tau_a = 50\text{ MPa}$ $\quad \sigma_{ba} = 110\text{ MPa}$

$t_A = 40\text{ mm}$ $\quad t_B = 40\text{ mm}$

$t_C = 50$ $\quad d_{pA} = 25\text{ mm}$

$d_{pB} = 30$ $\quad d_{pC} = 22\text{ mm}$

(a) RESULTANT FORCES F ACTING ON PULLEYS A, B, AND C

$F_A = \sqrt{2}\,T \qquad F_B = 2T$

$F_C = T \qquad T = \dfrac{Mg}{2} + \dfrac{W_{max}}{2}$

$W_{max} = 2T - Mg$

From statics at B

(b) MAXIMUM LOAD W THAT CAN BE ADDED AT B DUE TO τ_a AND σ_{ba} IN PINS AT A, B, AND C

PULLEY AT A

$\tau_A = \dfrac{F_A}{A_s}$

DOUBLE SHEAR

$F_A = \tau_a A_s \qquad \sqrt{2}T = \tau_a A_s$

$$\frac{Mg}{2} + \frac{W_{max}}{2} = \frac{\tau_a A_s}{\sqrt{2}}$$

$$W_{max1} = \frac{2}{\sqrt{2}}\left(\tau_a A_s\right) - Mg$$

$$W_{max1} = \frac{2}{\sqrt{2}}\left(\tau_a 2\frac{\pi}{4} d_p A^2\right) - Mg$$

$$\frac{W_{max1}}{Mg} = 22.6$$

$W_{max1} = 66.5\text{ kN}$ $\leftarrow$

(shear at A controls)

OR check bearing stress

$$W_{max2} = \frac{2}{\sqrt{2}}\left(\sigma_{ba} A_b\right) - Mg$$

$$W_{max2} = \frac{2}{\sqrt{2}}\left(\sigma_{ba} t_A d_{pA}\right) - Mg$$

$W_{max2} = 152.6\text{ kN}$ (bearing at A)

PULLEY AT B $\qquad 2T = \tau_a A_s$

$$W_{max3} = \frac{2}{2}(\tau_a A_s) - Mg$$

$$W_{max3} = \left[\tau_a\left(2\frac{\pi}{4} d_{pB}{}^2\right)\right] - Mg$$

$W_{max3} = 67.7\text{ kN}$ (shear at B)

$$W_{max4} = \frac{2}{2}(\sigma_{ba} A_b) - Mg$$

$$W_{max4} = \sigma_{ba} t_B d_{pB} - Mg$$

$W_{max4} = 129.1\text{ kN}$ (bearing at B)

PULLEY AT C $\qquad T = \tau_a A_s$

$$W_{max5} = 2(\tau_a A_s) - Mg$$

$$W_{max5} = \left[2\tau_a\left(2\frac{\pi}{4} d_{pC}{}^2\right)\right] - Mg$$

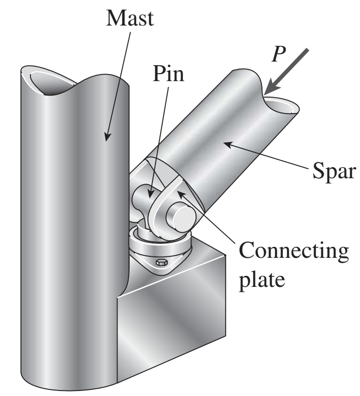
$W_{max5} = 7.3 \times 10^4$ $\quad W_{max5} = 73.1\text{ kN}$ (shear at C)

$$W_{max6} = 2\sigma_{ba} t_C d_{pC} - Mg$$

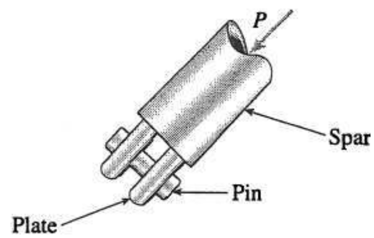
$W_{max6} = 239.1\text{ kN}$ (bearing at C)

Problem 1.8-9 A ship's spar is attached at the base of a mast by a pin connection (see figure). The spar is a steel tube of outer diameter $d_2 = 3.5$ in. and inner diameter $d_1 = 2.8$ in. The steel pin has diameter $d = 1$ in., and the two plates connecting the spar to the pin have thickness $t = 0.5$ in. The allowable stresses are as follows: compressive stress in the spar, 10 ksi; shear stress in the pin, 6.5 ksi; and bearing stress between the pin and the connecting plates, 16 ksi.

Determine the allowable compressive force P_{allow} in the spar.



Solution 1.8-9



Numerical data

$d_2 = 3.5$ in. $\quad d_1 = 2.8$ in.

$d_p = 1$ in. $\quad t = 0.5$ in.

$\sigma_a = 10$ ksi $\quad \tau_a = 6.5$ ksi $\quad \sigma_{ba} = 16$ ksi

Compressive stress in spar

$$P_{a1} = \sigma_a \frac{\pi}{4}\left(d_2^{\,2} - d_1^{\,2}\right) \qquad P_{a1} = 34.636 \text{ k}$$

Shear stress in pin

$$P_{a2} = \tau_a\left(2\frac{\pi}{4}\,d_p^{\,2}\right)$$

$$P_{a2} = 10.21 \text{ kips} < \text{controls} \qquad \leftarrow$$

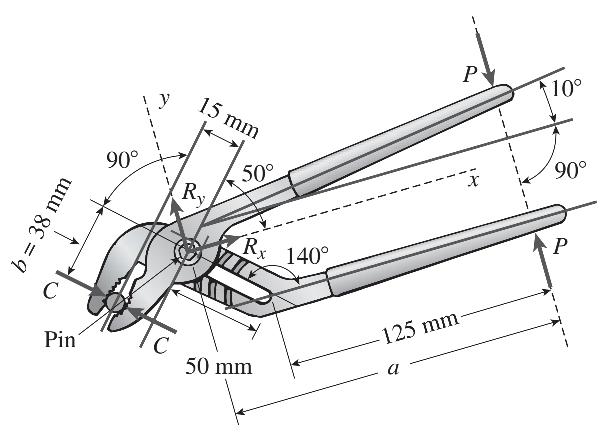
^double shear

Bearing stress between pin and conecting plates

$$P_{a3} = \sigma_{ba}(2d_p t) \qquad P_{a3} = 16 \text{ k}$$

Problem 1.8-10 What is the maximum possible value of the clamping force C in the jaws of the pliers shown in the figure if the ultimate shear stress in the 5-mm diameter pin is 340 MPa?

What is the maximum permissible value of the applied load P if a factor of safety of 3.0 with respect to failure of the pin is to be maintained?



Solution 1.8-10

NUMERICAL DATA

$FS = 3 \qquad \tau_u = 340 \text{ MPa} \qquad \tau_a = \dfrac{\tau_u}{FS}$

$d = 5 \text{ mm}$

$\tau_a = \dfrac{\sqrt{R_x^{\,2} + R_y^{\,2}}}{A_s}$ < pin at C in single shear

$R_x = -C\cos(40°) \qquad R_y = P + C\sin(40°)$

$a = 50\cos(40°) + 125 \qquad a = 163.302 \text{ mm}$

$b = 38 \text{ mm}$

STATICS $\quad \sum M_{pin} = 0 \qquad C = \dfrac{P(a)}{b}$

$R_x = -\dfrac{P(a)}{b}\cos(40°) \qquad R_y = P\left[1 + \dfrac{a}{b}\sin(40°)\right]$

$$P\sqrt{\left[-\frac{a}{b}\cos(40°)\right]^2 + \left[1 + \frac{a}{b}\sin(40°)\right]^2} = \tau_a A_s$$

$A_s = \dfrac{\pi}{4}d^2$

$\tau_a = \dfrac{\tau_u}{FS} \qquad \tau_a = 113.333 \text{ MPa}$

Find P_{max}

$$P_{max} = \frac{\tau_a A_s}{\sqrt{\left[-\frac{a}{b}\cos(40°)\right]^2 + \left[1 + \frac{a}{b}\sin(40°)\right]^2}}$$

$P_{max} = 445 \text{ N} \qquad \leftarrow$

here $\dfrac{a}{b} = 4.297$ < a/b = **mechanical advantage**

FIND MAXIMUM CLAMPING FORCE

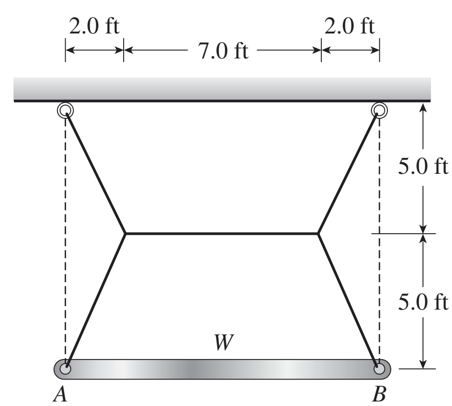
$C_{ult} = P_{max}FS\left(\dfrac{a}{b}\right) \qquad C_{ult} = 5739 \text{ N} \qquad \leftarrow$

$P_{ult} = P_{max}FS \qquad P_{ult} = 1335$

$\dfrac{C_{ult}}{P_{ult}} = 4.297$

Problem 1.8-11 A metal bar AB of weight W is suspended by a system of steel wires arranged as shown in the figure. The diameter of the wires is 5/64 in., and the yield stress of the steel is 65 ksi.

Determine the maximum permissible weight W_{max} for a factor of safety of 1.9 with respect to yielding.



Solution 1.8-11

NUMERICAL DATA

$d = \dfrac{5}{64} \text{ in.} \qquad \sigma_Y = 65 \text{ ksi} \qquad FS_y = 1.9$

$\sigma_a = \dfrac{\sigma_Y}{FS_y} \qquad \sigma_a = 34.211 \text{ ksi}$

FORCES IN WIRES AC, EC, BD, AND FD

$\sum F_V = 0$ at A, B, E, or F

$F_W = \dfrac{\sqrt{2^2 + 5^2}}{5} \times \dfrac{W}{2} \qquad \dfrac{\sqrt{2^2 + 5^2}}{10} = 0.539$

$W_{max} = 0.539\,\sigma_a \times A$

$$W_{max} = 0.539\left(\frac{\sigma_Y}{FS_y}\right)\left(\frac{\pi}{4}d^2\right)$$

$$W_{max} = 0.305 \text{ kips} \quad \leftarrow$$

CHECK ALSO FORCE IN WIRE CD

$\sum F_H = 0$ at C or D

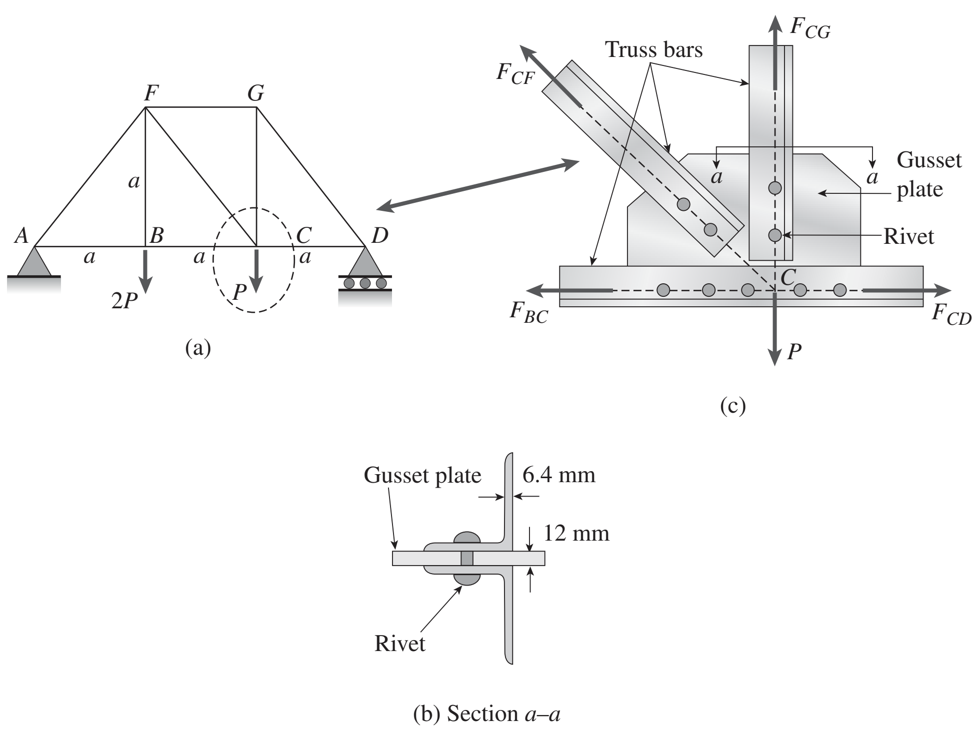
$$F_{CD} = 2\left(\frac{2}{\sqrt{2^2 + 5^2}} F_w\right)$$

$$F_{CD} = 2\left[\frac{2}{\sqrt{2^2 + 5^2}}\left(\frac{\sqrt{2^2 + 5^2}}{5} \times \frac{W}{2}\right)\right]$$

$$F_{CD} = \frac{2}{5} W \quad \text{less than } F_{AC} \text{ so } AC \text{ controls}$$

Problem 1.8-12 A plane truss is subjected to loads $2P$ and P at joints B and C, respectively, as shown in the figure part (a). The truss bars are made of two L102 × 76 × 6.4 steel angles [see Table F-5(b): cross sectional area of the two angles, $A = 2180 \text{ mm}^2$, figure part (b)] having an ultimate stress in tension equal to 390 MPa. The angles are connected to an 12 mm-thick gusset plate at C [figure part (c)] with 16-mm diameter rivets; assume each rivet transfers an equal share of the member force to the gusset plate. The ultimate stresses in shear and bearing for the rivet steel are 190 MPa and 550 MPa, respectively.

Determine the allowable load P_{allow} if a safety factor of 2.5 is desired with respect to the ultimate load that can be carried. (Consider tension in the bars, shear in the rivets, bearing between the rivets and the bars, and also bearing between the rivets and the gusset plate. Disregard friction between the plates and the weight of the truss itself.)



Solution 1.8-12

NUMERICAL DATA

$A = 2180\ \text{mm}^2$

$t_g = 12\ \text{mm}$ $\quad d_r = 16\ \text{mm}$ $\quad t_{\text{ang}} = 6.4\ \text{mm}$

$\sigma_u = 390\ \text{MPa}$ $\quad \tau_u = 190\ \text{MPa}$

$\sigma_{bu} = 550\ \text{MPa}$ $\quad \text{FS} = 2.5$

$\sigma_a = \dfrac{\sigma_u}{\text{FS}}$ $\quad \tau_a = \dfrac{\tau_u}{\text{FS}}$ $\quad \sigma_{ba} = \dfrac{\sigma_{bu}}{\text{FS}}$

MEMBER FORCES FROM TRUSS ANALYSIS

$F_{BC} = \dfrac{5}{3}P$ $\quad F_{CD} = \dfrac{4}{3}P$ $\quad F_{CF} = \dfrac{\sqrt{2}}{3}P$

$\dfrac{\sqrt{2}}{3} = 0.471$ $\quad F_{CG} = \dfrac{4}{3}P$

P_{allow} FOR TENSION ON NET SECTION IN TRUSS BARS

$A_{\text{net}} = A - 2d_r t_{\text{ang}}$ $\quad A_{\text{net}} = 1975\ \text{mm}^2$

$\dfrac{A_{\text{net}}}{A} = 0.906$

$F_{\text{allow}} = \sigma_a A_{\text{net}}$ $\quad <$ allowable force in a member so BC controls since it has the largest member force for this loading

$P_{\text{allow}} = \dfrac{3}{5}F_{BC\max}$ $\quad P_{\text{allow}} = \dfrac{3}{5}(\sigma_a A_{\text{net}})$

$P_{\text{allow}} = 184.879\ \text{kN}$

Next, P_{allow} for shear in rivets (all are in double shear)

$A_s = 2\dfrac{\pi}{4}d_r^2$ $\quad <$ for one rivet in DOUBLE shear

$\dfrac{F_{\max}}{N} = \tau_a A_s$ $\quad N$ = number of rivets in a particular member (see drawing of connection detail)

$P_{BC} = 3\left(\dfrac{3}{5}\right)(\tau_a A_s)$ $\qquad P_{BC} = 55.0\ \text{kN}$

$P_{CF} = 2\left(\dfrac{3}{\sqrt{2}}\right)(\tau_a A_s)$ $\qquad P_{CF} = 129.7\ \text{kN}$

$P_{CG} = 2\left(\dfrac{3}{4}\right)(\tau_a A_s)$

$P_{CG} = 45.8\ \text{kN} \leftarrow$ $\quad <$ so shear in rivets in CG and CD controls P_{allow} here

$P_{CD} = 2\left(\dfrac{3}{4}\right)(\tau_a A_s)$ $\qquad P_{CD} = 45.8\ \text{kN} \quad \leftarrow$

Next, P_{allow} for bearing of rivets on truss bars $A_b = 2d_r t_{\text{ang}}$ $\quad <$ rivet bears on each angle in two angle pairs

$\dfrac{F_{\max}}{N} = \sigma_{ba} A_b$

$P_{BC} = 3\left(\dfrac{3}{5}\right)(\sigma_{ba} A_b)$ $\qquad P_{BC} = 81.101\ \text{kN}$

$P_{CF} = 2\left(\dfrac{3}{\sqrt{2}}\right)(\sigma_{ba} A_b)$ $\qquad P_{CF} = 191.156\ \text{kN}$

$P_{CG} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\qquad P_{CG} = 67.584\ \text{kN}$

$P_{CD} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\qquad P_{CD} = 67.584\ \text{kN}$

Finally, P_{allow} for bearing of rivets on gusset plate

$A_b = d_r t_g$

(bearing area for each rivert on gusset plate)

$t_g = 12\ \text{mm} < 2t_{\text{ang}} = 12.8\ \text{mm}$

so gusset will control over angles

$P_{BC} = 3\left(\dfrac{3}{5}\right)(\sigma_{ba} A_b)$ $\qquad P_{BC} = 76.032\ \text{kN}$

$P_{CF} = 2\left(\dfrac{3}{\sqrt{2}}\right)(\sigma_{ba} A_b)$ $\qquad P_{CF} = 179.209\ \text{kN}$

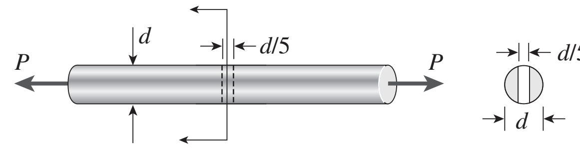
$P_{CG} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\qquad P_{CG} = 63.36\ \text{kN}$

$P_{CD} = 2\left(\dfrac{3}{4}\right)(\sigma_{ba} A_b)$ $\qquad P_{CD} = 63.36\ \text{kN}$

So, shear in rivets controls: $P_{\text{allow}} = 45.8\ \text{kN} \quad \leftarrow$

Problem 1.8-13 A solid bar of circular cross section (diameter d) has a hole of diameter $d/5$ drilled laterally through the center of the bar (see figure). The allowable average tensile stress on the net cross section of the bar is σ_{allow}.

(a) Obtain a formula for the allowable load P_{allow} that the bar can carry in tension.
(b) Calculate the value of P_{allow} if the bar is made of brass with diameter $d = 1.75$ in. and $\sigma_{\text{allow}} = 12$ ksi.
(*Hint*: Use the formulas of Case 15 Appendix E.)



Solution 1.8-13

NUMERICAL DATA

$d = 1.75 \text{ in.} \qquad \sigma_a = 12 \text{ ksi}$

(a) FORMULA FOR P_{ALLOW} IN TENSION

From Case 15, Appendix E:

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right) \qquad r = \frac{d}{2} \qquad a = \frac{d}{10}$$

$$\alpha = a\cos\left(\frac{a}{r}\right) \qquad r = 0.875 \text{ in.} \qquad a = 0.175 \text{ in.}$$

$$\alpha\frac{180}{\pi} = 78.463°$$

$$b = \sqrt{r^2 - a^2}$$

$$b = \sqrt{\left[\left(\frac{d}{2}\right)^2 - \left(\frac{d}{10}\right)^2\right]}$$

$$b = \sqrt{\left(\frac{6}{25}d^2\right)} \qquad b = \frac{d}{5}\sqrt{6}$$

$$P_a = \sigma_a A$$

$$P_a = \sigma_a\left[\frac{1}{2}d^2\left(\arccos\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}\right)\right]$$

$$\frac{\arccos\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}}{2} = 0.587 \qquad \frac{\pi}{4} = 0.785$$

$$P_a = \sigma_a(0.587\,d^2) \quad \leftarrow$$

$$\frac{0.587}{0.785} = 0.748$$

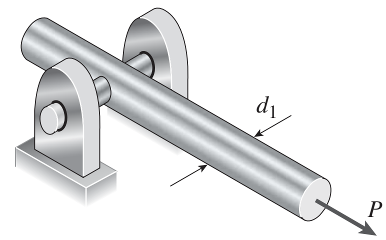
(b) EVALUATE NUMERICAL RESULT

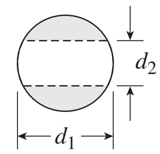
$d = 1.75 \text{ in.} \qquad \sigma_a = 12 \text{ ksi}$

$P_a = 21.6 \text{ k} \quad \leftarrow$

Problem 1.8-14 A solid steel bar of diameter $d_1 = 60$ mm has a hole of diameter $d_2 = 32$ mm drilled through it (see figure). A steel pin of diameter d_2 passes through the hole and is attached to supports.

Determine the maximum permissible tensile load P_{allow} in the bar if the yield stress for shear in the pin is $\tau_Y = 120$ MPa, the yield stress for tension in the bar is $\sigma_Y = 250$ MPa and a factor of safety of 2.0 with respect to yielding is required. (*Hint*: Use the formulas of Case 15, Appendix E.)





Solution 1.8-14

NUMERICAL DATA

$d_1 = 60$ mm $\quad d_2 = 32$ mm

$\tau_Y = 120$ MPa $\quad \sigma_Y = 250$ MPa

$FS_y = 2$

ALLOWABLE STRESSES

$$\tau_a = \frac{\tau_Y}{FS_y} \qquad \tau_a = 60 \text{ MPa}$$

$$\sigma_a = \frac{\sigma_Y}{FS_y} \qquad \sigma_a = 125 \text{ MPa}$$

From Case 15, Appendix E: $\quad r = \dfrac{d_1}{2}$

$$A = 2r^2\left(\alpha - \frac{ab}{r^2}\right) \qquad \alpha = \text{arc cos}\,\frac{d_2/2}{d_1/2} = \text{arc cos}\,\frac{d_2}{d_1}$$

$$a = \frac{d_2}{2} \qquad b = \sqrt{r^2 - a^2}$$

SHEAR AREA (DOUBLE SHEAR)

$$A_s = 2\left(\frac{\pi}{4}d_2{}^2\right) \qquad A_s = 1608 \text{ mm}^2$$

NET AREA IN TENSION (FROM APPENDIX E)

$$A_{net} = 2\left(\frac{d_1}{2}\right)^2 \left[\arccos\left(\frac{d_2}{d_1}\right) - \frac{\dfrac{d_2}{2}\left[\sqrt{\left(\dfrac{d_1}{2}\right)^2 - \left(\dfrac{d_2}{2}\right)^2}\right]}{\left(\dfrac{d_1}{2}\right)^2}\right]$$

$A_{net} = 1003 \text{ mm}^2$

P_{allow} in tension: smaller of values based on either shear or tension allowable stress *x* appropriate area

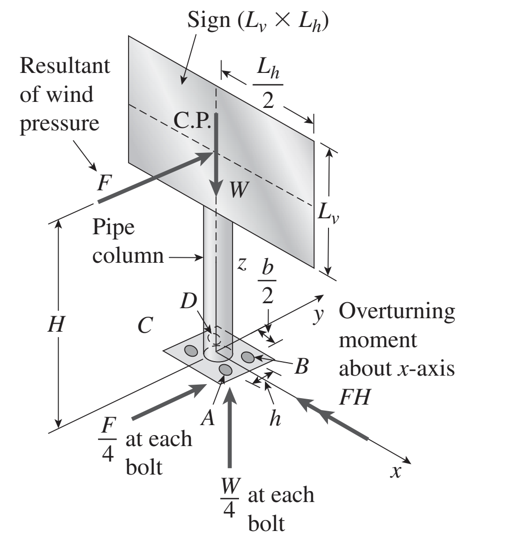
$P_{a1} = \tau_a A_s \quad P_{a1} = 96.5 \text{ kN} <$ shear governs $\leftarrow$

$P_{a2} = \sigma_a A_{net} \qquad P_{a2} = 125.4 \text{ kN}$

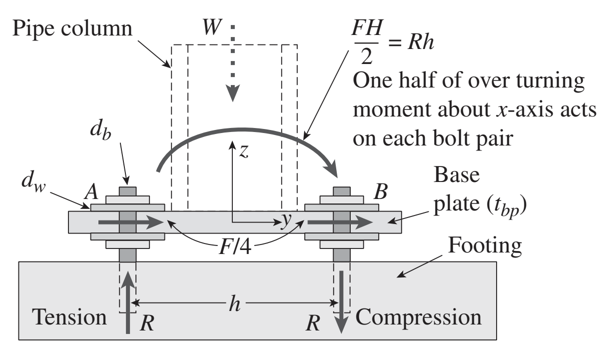
Problem 1.8-15 A sign of weight W is supported at its base by four bolts anchored in a concrete footing. Wind pressure p acts normal to the surface of the sign; the resultant of the uniform wind pressure is force F at the center of pressure. The wind force is assumed to create equal shear forces $F/4$ in the y-direction at each bolt [see figure parts (a) and (c)]. The overturning effect of the wind force also causes an uplift force R at bolts A and C and a downward force $(-R)$ at bolts B and D [see figure part (b)]. The resulting effects of the wind, and the associated ultimate stresses for each stress condition, are: normal stress in each bolt ($\sigma_u = 60$ ksi); shear through the base plate ($\tau_u = 17$ ksi); horizontal shear and bearing on each bolt ($\tau_{hu} = 25$ ksi and $\sigma_{bu} = 75$ ksi); and bearing on the bottom washer at B (or D) ($\sigma_{bw} = 50$ ksi).

Find the maximum wind pressure $p_{\max}$ (psf) that can be carried by the bolted support system for the sign if a safety factor of 2.5 is desired with respect to the ultimate wind load that can be carried.

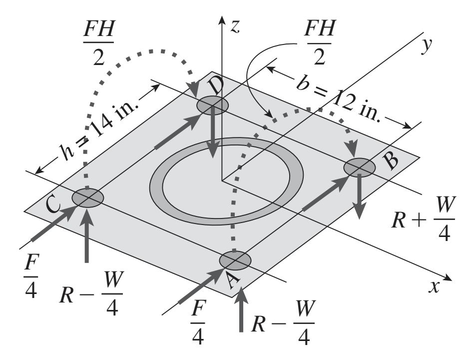
Use the following numerical data: bolt $d_b = 3/4$ in.; washer $d_w = 1.5$ in.; base plate $t_{bp} = 1$ in.; base plate dimensions $h = 14$ in. and $b = 12$ in.; $W = 500$ lb; $H = 17$ ft; sign dimensions ($L_v = 10$ ft. $\times$ $L_h = 12$ ft.); pipe column diameter $d = 6$ in., and pipe column thickness $t = 3/8$ in.



(a)



(b)



(c)

Solution 1.8-15

NUMERICAL DATA

$\sigma_u = 60 \text{ ksi} \qquad \tau_u = 17 \text{ ksi} \qquad \tau_{hu} = 25 \text{ ksi}$

$\sigma_{bu} = 75 \text{ ksi} \qquad \sigma_{bw} = 50 \text{ ksi} \qquad FS_u = 2.5$

$d_b = \frac{3}{4} \text{ in.} \qquad d_w = 1.5 \text{ in.} \qquad t_{bp} = 1 \text{ in.}$

$h = 14 \text{ in.} \qquad b = 12 \text{ in.} \qquad d = 6 \text{ in.} \qquad t = \frac{3}{8} \text{ in.}$

$W = 0.500 \text{ kips} \qquad H = 17(12) \qquad H = 204 \text{ in.}$

$L_v = 10(12) \qquad L_h = 12(12) \qquad L_v = 120 \text{ in.}$

$L_h = 144 \text{ in.}$

ALLOWABLE STRESSES (ksi)

$\sigma_a = \frac{\sigma_u}{FS_u} \qquad \sigma_a = 24 \qquad \tau_a = \frac{\tau_u}{FS_u}$

$\tau_a = 6.8 \qquad \tau_{ha} = \frac{\tau_{hu}}{FS_u} \qquad \tau_{ha} = 10$

$\sigma_{ba} = \frac{\sigma_{bu}}{FS_u} \qquad \sigma_{ba} = 30 \qquad \sigma_{bwa} = \frac{\sigma_{bw}}{FS_u}$

$\sigma_{bwa} = 20$

FORCES F AND R IN TERMS OF p_{max}

$F = p_{max} L_v L_h \qquad R = \frac{FH}{2h}$

$R = p_{max} \frac{L_v L_h H}{2h}$

(1) COMPUTE p_{max} BASED ON NORMAL STRESS IN EACH BOLT (GREATER AT B AND D)

$$\sigma = \frac{R + \frac{W}{4}}{\frac{\pi}{4} d_b^2} \qquad R_{max} = \sigma_a\left(\frac{\pi}{4} d_b^2\right) - \frac{W}{4}$$

$$p_{max1} = \frac{\sigma_a\left(\frac{\pi}{4} d_b^2\right) - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$p_{max1} = 11.98 \text{ psf} \quad \leftarrow \quad$ controls

(2) COMPUTE p_{max} BASED ON SHEAR THROUGH BASE PLATE (GREATER AT B AND D)

$$\tau = \frac{R + \frac{W}{4}}{\pi d_w t_{bp}}$$

$$R_{max} = \tau_a(\pi d_w t_{bp}) - \frac{W}{4}$$

$$p_{max2} = \frac{\tau_a(\pi d_w t_{bp}) - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$p_{max2} = 36.5 \text{ psf}$

(3) COMPUTE p_{max} BASED ON HORIZONTAL SHEAR ON EACH BOLT

$$\tau_h = \frac{\frac{F}{4}}{\left(\frac{\pi}{4} d_b^2\right)} \qquad F_{max} = 4\tau_{ha}\left(\frac{\pi}{4} d_b^2\right)$$

$$p_{max3} = \frac{\tau_{ha}(\pi d_b^2)}{L_v L_h}$$

$$p_{max3} = 147.3 \text{ psf}$$

(4) COMPUTE p_{max} BASED ON HORIZONTAL BEARING ON EACH BOLT

$$\sigma_b = \frac{\frac{F}{4}}{(t_{bp} d_b)} \qquad F_{max} = 4\sigma_{ba}(t_{bp} d_b)$$

$$p_{max4} = \frac{4\sigma_{ba}(t_b p d_b)}{L_v L_h}$$

$$p_{max4} = 750 \text{ psf}$$

(5) COMPUTE p_{max} BASED ON BEARING UNDER THE TOP WASHER AT A (OR C) AND THE BOTTOM WASHER AT B (OR D)

$$\sigma_{bw} = \frac{R + \frac{W}{4}}{\frac{\pi}{4}\left(d_w^2 - d_b^2\right)}$$

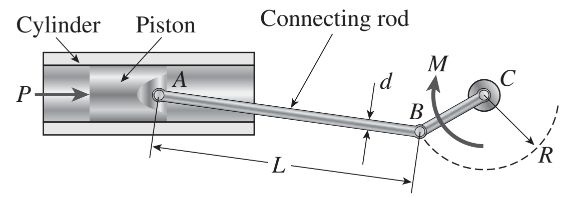
$$R_{max} = \sigma_{bwa}\left[\frac{\pi}{4}\left(d_w^2 - d_b 2\right)\right] - \frac{W}{4}$$

$$p_{max5} = \frac{\sigma_{bwa}\left[\frac{\pi}{4}\left(d_w^2 - d_b^2\right)\right] - \frac{W}{4}}{\frac{L_v L_h H}{2h}}$$

$$p_{max5} = 30.2 \text{ psf}$$

So, normal/stress in bolts controls; $p_{max} = 11.98$ psf

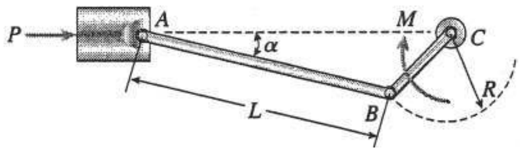
Problem 1.8-16 The piston in an engine is attached to a connecting rod AB, which in turn is connected to a crank arm BC (see figure). The piston slides without friction in a cylinder and is subjected to a force P (assumed to be constant) while moving to the right in the figure. The connecting rod, which has diameter d and length L, is attached at both ends by pins. The crank arm rotates about the axle at C with the pin at B moving in a circle of radius R. The axle at C, which is supported by bearings, exerts a resisting moment M against the crank arm.



(a) Obtain a formula for the maximum permissible force P_{allow} based upon an allowable compressive stress σ_c in the connecting rod.

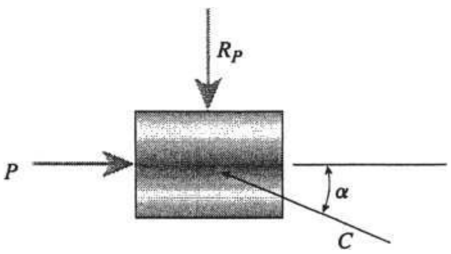
(b) Calculate the force P_{allow} for the following data: $\sigma_c = 160$ MPa, $d = 9.00$ mm, and $R = 0.28L$.

Solution 1.8-16



d = diameter of rod AB

FREE-BODY DIAGRAM OF PISTON



P = applied force (constant)
C = compressive force in connecting rod
RP = resultant of reaction forces between cylinder and piston (no friction)

$\sum F_{\text{horiz}} = 0 \overset{\rightarrow}{+} \overset{\leftarrow}{-}$

$P - C \cos \alpha = 0$

$P = C \cos \alpha$

MAXIMUM COMPRESSIVE FORCE C IN CONNECTING ROD

$C_{\max} = \sigma_c A_c$

in which A_c = area of connecting rod

$$A_c = \frac{\pi d^2}{4}$$

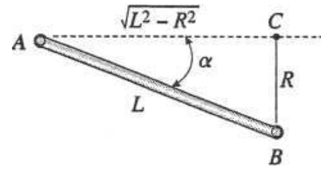
MAXIMUM ALLOWABLE FORCE P

$P = C_{\max} \cos \alpha$

$= \sigma_c A_c \cos \alpha$

The maximun allowable force P occurs when $\cos \alpha$ has its smallest value, which means that α has its largest value.

LARGEST VALUE OF α



The largest value of α occurs when point B is the farthest distance from line AC. The farthest distance is the radius R of the crank arm.

Therefore,

$\overline{BC} = R$

Also, $\overline{AC} = \sqrt{L^2 - R^2}$

$$\cos \alpha = \frac{\sqrt{L^2 - R^2}}{L} = \sqrt{1 - \left(\frac{R}{L}\right)^2}$$

(a) MAXIMUM ALLOWABLE FORCE P

$P_{\text{allow}} = \sigma_c A_c \cos \alpha$

$$= \sigma_c \left(\frac{\pi d^2}{4}\right) \sqrt{1 - \left(\frac{R}{L}\right)^2} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES

$\sigma_c = 160$ MPa $\quad d = 9.00$ mm

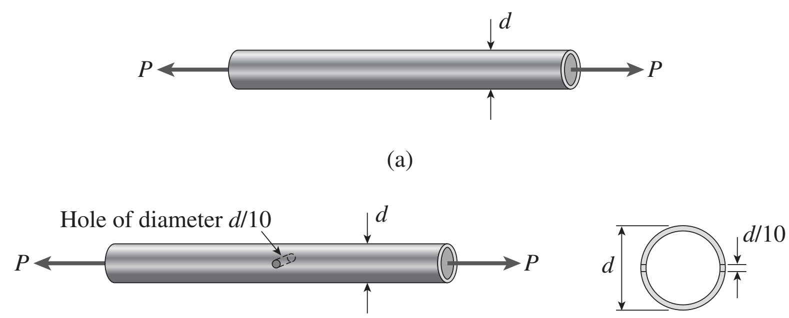
$R = 0.28L \quad R/L = 0.28$

$P_{\text{allow}} = 9.77$ kN $\quad \leftarrow$

Design for Axial Loads and Direct Shear

Problem 1.9-1 An aluminum tube is required to transmit an axial tensile force $P = 33$ k [see figure part (a)]. The thickness of the wall of the tube is to be 0.25 in.

(a) What is the minimum required outer diameter d_{min} if the allowable tensile stress is 12,000 psi?
(b) Repeat part (a) if the tube will have a hole of diameter $d/10$ at mid-length [see figure parts (b) and (c)].



(a) (b) (c)

Solution 1.9-1

NUMERICAL DATA

$P = 33$ kips $\qquad t = 0.25$ in. $\qquad \sigma_a = 12$ ksi

(a) MINIMUM DIAMETER OF TUBE (NO HOLES)

$$A_1 = \frac{\pi}{4}\left[d^2-(d-2t)^2\right] \qquad A_2 = \frac{P}{\sigma_a}$$

$$A_2 = 2.75 \text{ in.}^2$$

Equating A_1 and A_2 and solving for d:

$$d = \frac{P}{\pi\sigma_a t} + t \qquad d = 3.75 \text{ in.} \quad \leftarrow$$

(b) MINIMUM DIAMETER OF TUBE (WITH HOLES)

$$A_1 = \left[\frac{\pi}{4}\left[d^2-(d-2t)^2\right]-2\left(\frac{d}{10}\right)t\right]$$

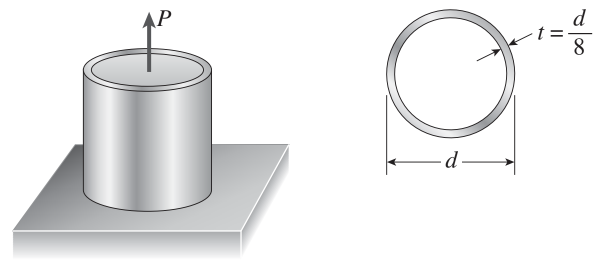
$$A_1 = d\left(\pi t - \frac{t}{5}\right) - \pi t^2$$

Equating A_1 and A_2 and solving for d:

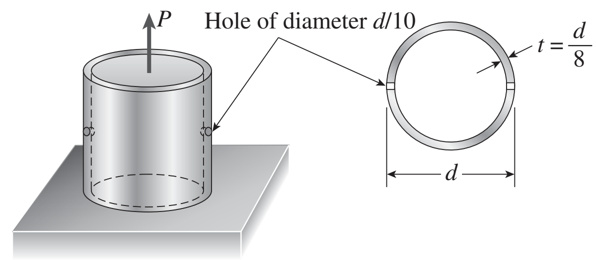
$$d = \frac{\dfrac{P}{\sigma_a} + \pi t^2}{\pi t - \dfrac{t}{5}} \qquad d = 4.01 \text{ in.} \quad \leftarrow$$

Problem 1.9-2 A copper alloy pipe having yield stress $\sigma_Y = 290$ MPa is to carry an axial tensile load $P = 1500$ kN [see figure part (a)]. A factor of safety of 1.8 against yielding is to be used.

(a) If the thickness t of the pipe is to be one-eighth of its outer diameter, what is the minimum required outer diameter d_{min}?
(b) Repeat part (a) if the tube has a hole of diameter $d/10$ drilled through the entire tube as shown in the figure [part (b)].



(a)



(b)

Solution 1.9-2

Numerical data

$\sigma_Y = 290$ MPa

$P = 1500$ kN

$FS_y = 1.8$

(a) Minimum diameter (no holes)

$$A_1 = \frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{4}\right)^2\right]$$

$$A_1 = \frac{7}{64}\pi d^2$$

$$A_2 = \frac{P}{\frac{\sigma_Y}{FS_y}} \qquad A_2 = 9.31 \times 10^3\ \text{mm}^2$$

Equate A_1 and A_2 and solve for d:

$$d^2 = \frac{7}{64\pi}\left(\frac{P}{\frac{\sigma_Y}{FS_y}}\right)$$

$$d_{min}\sqrt{\frac{7}{64\pi}\left(\frac{P}{\frac{\sigma_Y}{FS_y}}\right)}$$

$$d_{min} = 164.6\ \text{mm} \quad \leftarrow$$

(b) MINIMUM DIAMETER (WITH HOLES)

Redefine A_1—subtract area for two holes—then equate to A_2

$$A_1 = \left[\frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{4}\right)^2\right] - 2\left(\frac{d}{10}\right)\left(\frac{d}{8}\right)\right]$$

$$A_1 = \frac{7}{64}\pi d^2 - \frac{1}{40}d^2$$

$$A_1 = d^2\left(\frac{7}{64}\pi - \frac{1}{40}\right) \qquad \frac{7}{64}\pi - \frac{1}{40} = 0.319$$

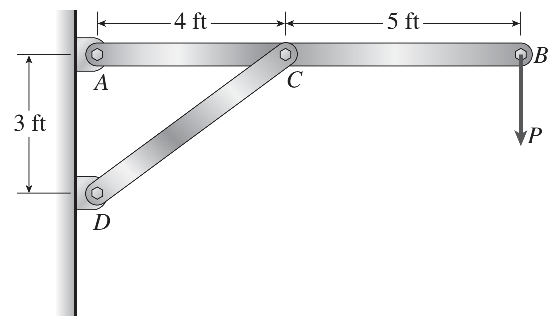
$$d^2 = \frac{\left(\dfrac{P}{\dfrac{\sigma y}{FS_y}}\right)}{\left(\dfrac{7}{64}\pi - \dfrac{1}{40}\right)}$$

$$d_{min} = \sqrt{\frac{\left(\dfrac{P}{\dfrac{\sigma_y}{FS_y}}\right)}{\left(\dfrac{7}{64}\pi - \dfrac{1}{40}\right)}} \qquad d_{min} = 170.9 \text{ mm} \quad \longleftarrow$$

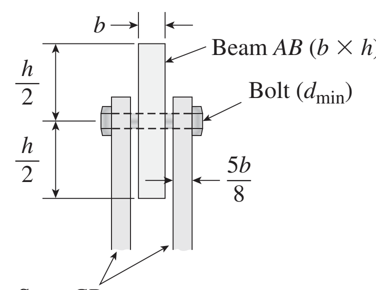
Problem 1.9-3 A horizontal beam AB with cross-sectional dimensions ($b = 0.75$ in.) $\times$ ($h = 8.0$ in.) is supported by an inclined strut CD and carries a load $P = 2700$ lb at joint B [see figure part (a)]. The strut, which consists of two bars each of thickness $5b/8$, is connected to the beam by a bolt passing through the three bars meeting at joint C [see figure part (b)].

(a) If the allowable shear stress in the bolt is 13,000 psi, what is the minimum required diameter d_{min} of the bolt at C?

(b) If the allowable bearing stress in the bolt is 19,000 psi, what is the minimum required diameter d_{min} of the bolt at C?



(a)



(b)

Solution 1.9-3

NUMERICAL DATA

$P = 2.7 \text{ k} \qquad b = 0.75 \text{ in.} \qquad h = 8 \text{ in.}$
$\tau_a = 13 \text{ ksi} \qquad \sigma_{ba} = 19 \text{ ksi}$

(a) d_{min} BASED ON ALLOWABLE SHEAR—DOUBLE SHEAR IN STRUT

$$\tau_a = \frac{F_{DC}}{A_s} \qquad F_{DC} = \frac{15}{4}P$$

$$A_s = 2\left(\frac{\pi}{4}d^2\right)$$

$$d_{min} = \sqrt{\frac{\frac{15}{4}P}{\tau_a\left(\frac{\pi}{2}\right)}} \qquad d_{min} = 0.704 \text{ in.} \quad \leftarrow$$

(b) d_{min} BASED ON ALLOWABLE BEARING AT JT C

Bearing from beam ACB $\qquad \sigma_b = \dfrac{15\,P/4}{b\,d}$

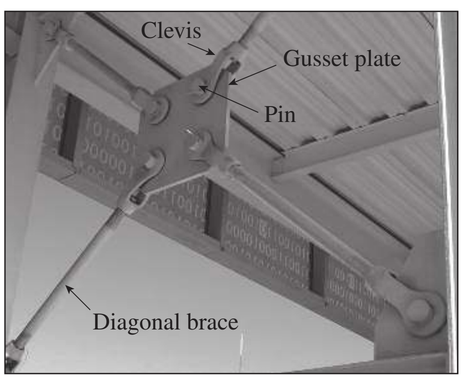
$$d_{min} = \frac{15\,P/4}{b\,\sigma_{ba}} \qquad d_{min} = 0.711 \text{ in.} \quad \leftarrow$$

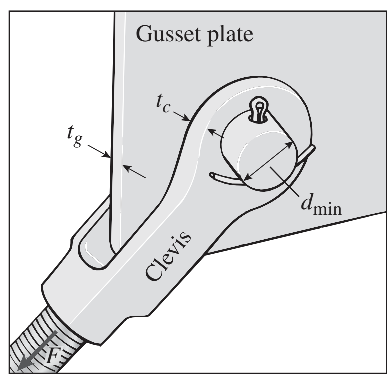
Bearing from strut DC $\quad \sigma_b = \dfrac{\frac{15}{4}P}{2\frac{5}{8}b\,d}$

$$\sigma_b = 3\frac{P}{b\,d} \qquad \text{(lower than } ACB\text{)}$$

Problem 1.9-4 Lateral bracing for an elevated pedestrian walkway is shown in the figure part (a). The thickness of the clevis plate t_c = 16 mm and the thickness of the gusset plate t_g = 20 mm [see figure part (b)]. The maximum force in the diagonal bracing is expected to be F = 190 kN.

If the allowable shear stress in the pin is 90 MPa and the allowable bearing stress between the pin and both the clevis and gusset plates is 150 MPa, what is the minimum required diameter d_{min} of the pin?





Solution 1.9-4

NUMERICAL DATA

$F = 190\text{ kN}$ $\tau_a = 90\text{ MPa}$ $\sigma_{ba} = 150\text{ MPa}$

$t_g = 20\text{ mm}$ $t_c = 16\text{ mm}$

(1) d_{min} BASED ON ALLOW SHEAR—DOUBLE SHEAR IN PIN

$$\tau = \frac{F}{A_s} \qquad A_s = 2\left(\frac{\pi}{4}d^2\right)$$

$$d_{min} = \sqrt{\frac{F}{\tau_a\left(\frac{\pi}{2}\right)}} \qquad d_{min} = 36.7\text{ mm}$$

(2) d_{min} BASED ON ALLOW BEARING IN GUSSET AND CLEVIS PLATES

Bearing on gusset plate

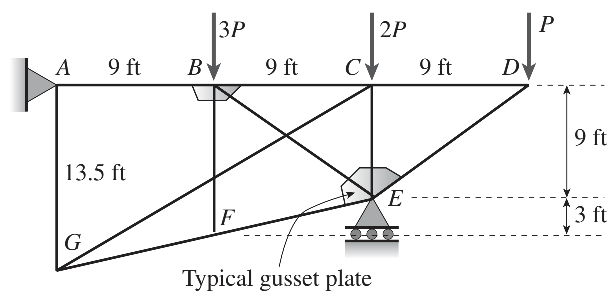
$$\sigma_b = \frac{F}{A_b} \qquad A_b = t_g d \qquad d_{min} = \frac{F}{t_g\sigma_{ba}}$$

$d_{min} = 63.3\text{ mm}$ <controls ←

Bearing on clevis $A_b = d(2t_c)$

$$d_{min} = \frac{F}{2t_c\sigma_{ba}} \qquad d_{min} = 39.6\text{ mm}$$

Problem 1.9-5 A plane truss has joint loads P, $2P$, and $3P$ at joints D, C, and B, respectively (see figure) where load variable $P = 5200$ lb. All members have two end plates (see figure for Prob. 1.7-2) which are pin connected to gusset plates (see also figure for Prob. 1.8-12). Each end plate has thickness $t_p = 0.625$ in., and all gusset plates have thickness $t_g = 1.125$ in. If the allowable shear stress in each pin is 12,000 psi and the allowable bearing stress in each pin is 18,000 psi, what is the minimum required diameter d_{min} of the pins used at either end of member BE?



Solution 1.9-5

$P = 5200\text{ lb}$ $F_{BE} = 3.83858\,P = 19{,}960.616\text{ lb}$ < from plane truss analysis (see Probs. 1.2-4 to 1.2-6) $\tau_a = 12\text{ ksi}$

$t_p = \frac{5}{8}\text{ in.}$ $t_g = 1.125\text{ in.}$ $t_p = 0.625\text{ in.}$ $2\,t_p = 1.25\text{ in.}$ $\sigma_{ba} = 18\text{ ksi}$

PIN DIAMETER BASED ON ALLOWABLE SHEAR STRESS (PINS IN DOUBLE SHEAR)

$$d_{p1} = \sqrt{\frac{\frac{F_{BE}}{2}}{\frac{\pi}{4}\tau_a}} = 1.029\text{ in.} \qquad < \text{controls} \qquad \boxed{d_{pin} = 1.029\text{ in.}}$$

PIN DIAMETER BASED ON BEARING BETWEEN PIN AND EACH OF TWO END PLATES < $2t_p$ is greater than t_g so gusset will control

$$d_{p2} = \frac{F_{BE}}{2\,t_p\sigma_{ba}} = 0.887\text{ in.}$$

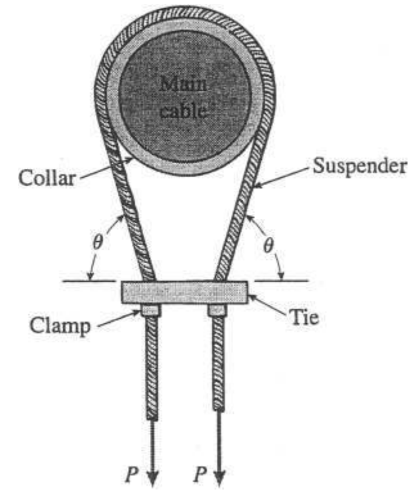
PIN DIAMETER BASED ON BEARING BETWEEN PIN AND GUSSET PLATE

$$d_{p3} = \frac{F_{BE}}{t_g\sigma_{ba}} = 0.986\text{ in.}$$

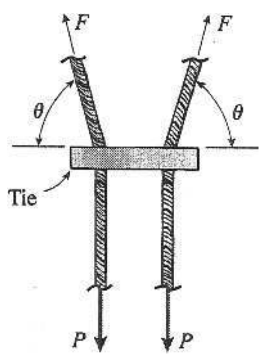
Problem 1.9-6 A suspender on a suspension bridge consists of a cable that passes over the main cable (see figure) and supports the bridge deck, which is far below. The suspender is held in position by a metal tie that is prevented from sliding downward by clamps around the suspender cable.

Let P represent the load in each part of the suspender cable, and let θ represent the angle of the suspender cable just above the tie. Finally, let σ_{allow} represent the allowable tensile stress in the metal tie.

(a) Obtain a formula for the minimum required cross-sectional area of the tie.
(b) Calculate the minimum area if $P = 130$ kN, $\theta = 75°$, and $\sigma_{allow} = 80$ MPa.



Solution 1.9-6 Suspender tie on a suspension bridge



F = tensile force in cable above tie

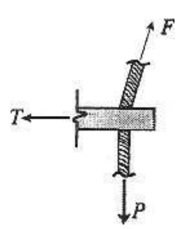
P = tensile force in cable below tie

σ_{allow} = allowable tensile stress in the tie

FREE-BODY DIAGRAM OF HALF THE TIE

Note: Include a small amount of the cable in the free-body diagram

T = tensile force in the tie



FORCE TRIANGLE

$$\cot\theta = \frac{T}{P}$$

$$T = P\cot\theta$$

(a) MINIMUM REQUIRED AREA OF TIE

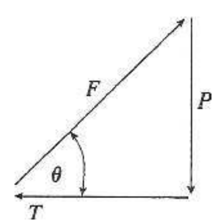
$$A_{min} = \frac{T}{\sigma_{allow}} = \frac{P\cot\theta}{\sigma_{allow}} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES:

$P = 130$ kN $\qquad \theta = 75°$

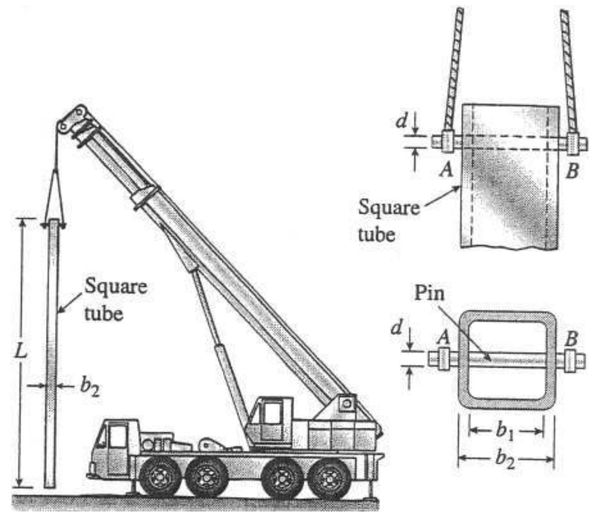
$\sigma_{allow} = 80$ MPa

$A_{min} = 435\ \text{mm}^2 \quad \leftarrow$

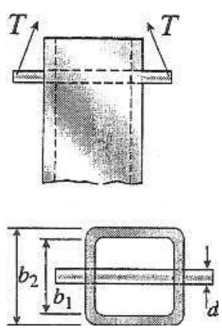


Problem 1.9-7 A square steel tube of length $L = 20$ ft and width $b_2 = 10.0$ in. is hoisted by a crane (see figure). The tube hangs from a pin of diameter d that is held by the cables at points A and B. The cross section is a hollow square with inner dimension $b_1 = 8.5$ in. and outer dimension $b_2 = 10.0$ in. The allowable shear stress in the pin is 8,700 psi, and the allowable bearing stress between the pin and the tube is 13,000 psi.

Determine the minimum diameter of the pin in order to support the weight of the tube. (*Note*: Disregard the rounded corners of the tube when calculating its weight.)



Solution 1.9-7 Tube hoisted by a crane



T = tensile force in cable

W = weight of steel tube

d = diameter of pin

b_1 = inner dimension of tube $= 8.5$ in.

b_2 = outer dimension of tube $= 10.0$ in.

L = length of tube = 20 ft

$\tau_{\text{allow}} = 8{,}700$ psi

$\sigma_b = 13{,}000$ psi

Weight of tube

γ_s = weight density of steel $= 490\ \text{lb/ft}^3$

A = area of tube

$$= b_2^2 - b_1^2 = (10.0\text{ in.})^2 - (8.5\text{ in.})^2 = 27.75\text{ in.}$$

$$W = \gamma_s AL = (490\ \text{lb/ft}^3)(27.75\ \text{in.}^2)\left(\frac{1\text{ft}^2}{144\ \text{in.}}\right)(20\text{ ft}) = 1{,}889\text{ lb}$$

Diameter of pin based upon shear

Double shear. $2\tau_{\text{allow}}A_{\text{pin}} = W$

$$2(8{,}700\text{ psi})\left(\frac{\pi\,d^2}{4}\right) = 1889\text{ lb}$$

$d^2 = 0.1382\ \text{in.}^2 \qquad d_1 = 0.372$ in.

Diameter of pin based upon bearing

$\sigma_b(b_2 - b_1)d = W$

$(13{,}000\text{ psi})(10.0\text{ in.} - 8.5\text{ in.})\,d = 1{,}889$ lb

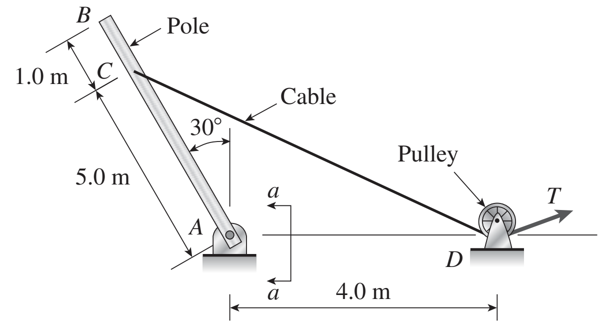
$d_2 = 0.097$ in.

Minimum diameter of pin

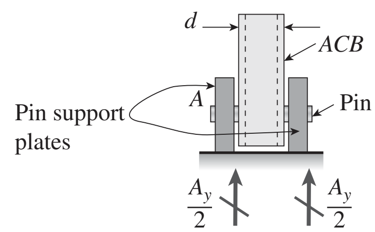
Shear governs. $\quad d_{\min} = 0.372$ in.

Problem 1.9-8 A cable and pulley system at D is used to bring a 230-kg pole (ACB) to a vertical position as shown in the figure part (a). The cable has tensile force T and is attached at C. The length L of the pole is 6.0 m, the outer diameter is d = 140 mm, and the wall thickness t = 12 mm. The pole pivots about a pin at A in figure part (b). The allowable shear stress in the pin is 60 MPa and the allowable bearing stress is 90 MPa.

Find the minimum diameter of the pin at A in order to support the weight of the pole in the position shown in the figure part (a).



(a)



(b)

Solution 1.9-8

ALLOWABLE SHEAR AND BEARING STRESSES

$\tau_a = 60$ MPa $\qquad \sigma_{ba} = 90$ MPa

FIND INCLINATION OF AND FORCE IN CABLE, T

let α = angle between pole and cable at C; use law of cosines

$$DC = \sqrt{5^2 + 4^2 - 2(5)(4)\cos\left(120\,\frac{\pi}{180}\right)}$$

$$DC = 7.81 \text{ m} \qquad \alpha = \arccos\left[\frac{5^2 + DC^2 - 4^2}{2DC(5)}\right]$$

$\alpha = 26.33°$ $\qquad \theta = 60 - \alpha$

$\theta = 33.67$ <ange between cable and horizontal at D

$W = 230 \text{ kg}(9.81 \text{ m/s}^2) \qquad W = 2.256 \times 10^3$ N

STATICS TO FIND CABLE FORCE T

$$\sum M_A = 0 \qquad W(3\sin(30°)) - T_X(5\cos(30°)) + T_y(5\sin(30°)) = 0$$

substitute for T_x and T_y in terms of T and solve for T:

$$T = \frac{\frac{3}{2}W}{\frac{-5}{2}\sin(\theta) + \frac{5\sqrt{3}}{2}\cos(\theta)}$$

$T = 1.53 \times 10^3$ N $\qquad T_x = T\cos(\theta)$

$T_y = T\sin(\theta) \qquad T_x = 1.27 \times 10^3$ N $\qquad T_y = 846.11$ N

(1) d_{min} BASED ON ALLOWABLE SHEAR–DOUBLE SHEAR AT A

$A_x = -T_x \qquad A_y = T_y + W$

CHECK SHEAR DUE TO RESULTANT FORCE ON PIN AT A

$$R_A = \sqrt{A_x^2 + A_y^2} \qquad R_A = 3.35 \times 10^3\ \text{N}$$

$$d_{min} = \sqrt{\frac{\frac{R_A}{2}}{\tau_a\left(\frac{\pi}{4}\right)}}$$

$d_{min} = 5.96$ mm <controls ←

(2) d_{min} BASED ON ALLOWABLE BEARING ON PIN

$d_{pole} = 140$ mm $\quad t_{pole} = 12$ mm
$L_{pole} = 6000$ mm

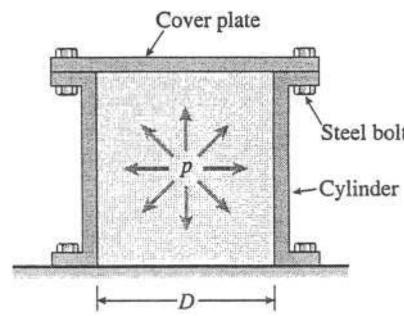
MEMBER AB BEARING ON PIN

$$\sigma_b = \frac{R_A}{A_b} \qquad A_b = 2t_{pole}d$$

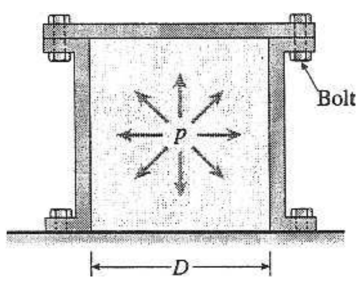
$$d_{min} = \frac{R_A}{2t_{pole}\sigma_{ba}} \qquad d_{min} = 1.55\ \text{mm}$$

Problem 1.9-9 A pressurized circular cylinder has a sealed cover plate fastened with steel bolts (see figure). The pressure p of the gas in the cylinder is 290 psi, the inside diameter D of the cylinder is 10.0 in., and the diameter d_B of the bolts is 0.50 in.

If the allowable tensile stress in the bolts is 10,000 psi, find the number n of bolts needed to fasten the cover.



Solution 1.9-9 Pressurized cylinder



$p = 290$ psi $\quad D = 10.0$ in. $\quad d_b = 0.50$ in.

$\sigma_{allow} = 10{,}000$ psi $\quad n$ = number of bolts

F = total force acting on the cover plate from the internal pressure

$$F = p\left(\frac{\pi D^2}{4}\right)$$

NUMBER OF BOLTS

P = tensile force in one bolt

$$P = \frac{F}{n} = \frac{\pi p D^2}{4n}$$

$$A_b = \text{area of one bolt} = \frac{\pi}{4}d_b^2$$

$$P = \sigma_{allow}A_b$$

$$\sigma_{allow} = \frac{P}{A_b} = \frac{\pi p D^2}{(4n)(\frac{\pi}{4})d_b^2} = \frac{pD^2}{nd_b^2}$$

$$n = \frac{pD^2}{d_b^2\sigma_{allow}}$$

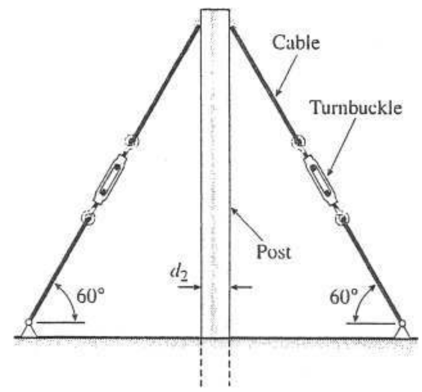
SUBSTITUTE NUMERICAL VALUES:

$$n = \frac{(290\ \text{psi})(10\ \text{in.})^2}{(0.5\ \text{in.})^2(10{,}000\ \text{psi})} = 11.6$$

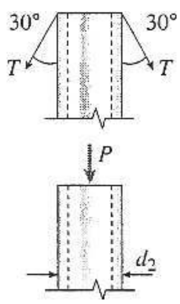
Use 12 bolts ←

Problem 1.9-10 A tubular post of outer diameter d_2 is guyed by two cables fitted with turnbuckles (see figure). The cables are tightened by rotating the turnbuckles, thus producing tension in the cables and compression in the post. Both cables are tightened to a tensile force of 110 kN. Also, the angle between the cables and the ground is 60°, and the allowable compressive stress in the post is σ_c = 35 MPa.

If the wall thickness of the post is 15 mm, what is the minimum permissible value of the outer diameter d_2?



Solution 1.9-10 Tubular post with guy cables



d_2 = outer diameter

d_1 = inner diameter

t = wall thickness = 15 mm

T = tensile force in a cable = 110 kN

σ_{allow} = 35 MPa

P = compressive force in post = $2T\cos 30°$

Required area of post

$$A = \frac{P}{\sigma_{\text{allow}}} = \frac{2T\cos 30°}{\sigma_{\text{allow}}}$$

Area of post

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[d_2^2 - (d_2 - 2t)^2]$$

$$= \pi t(d_2 - t)$$

Equate areas and solve for d_2:

$$\frac{2T\cos 30°}{\sigma_{\text{allow}}} = \pi t(d_2 - t)$$

$$d_2 = \frac{2T\cos 30°}{\pi t \sigma_{\text{allow}}} + t \quad \leftarrow$$

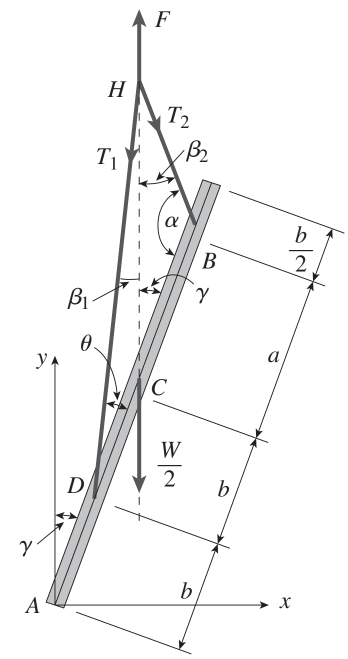
Substitute numerical values:

$(d_2)_{\text{min}}$ = 131 mm $\leftarrow$

Problem 1.9-11 A large precast concrete panel for a warehouse is being raised to a vertical position using two sets of cables at two lift lines as shown in the figure part (a). Cable 1 has length $L_1 = 22$ ft and distances along the panel (see figure part (b)) are $a = L_1/2$ and $b = L_1/4$. The cables are attached at lift points B and D and the panel is rotated about its base at A. However, as a worst case, assume that the panel is momentarily lifted off the ground and its total weight must be supported by the cables. Assuming the cable lift forces F at each lift line are about equal, use the simplified model of one half of the panel in figure part (b) to perform your analysis for the lift position shown. The total weight of the panel is $W = 85$ kips. The orientation of the panel is defined by the following angles: $\gamma = 20°$ and $\theta = 10°$.

Find the required cross-sectional area A_C of the cable if its breaking stress is 91 ksi and a factor of safety of 4 with respect to failure is desired.

(a)



(b)

Solution 1.9-11

GEOMETRY

$L_1 = 22$ ft $\quad a = \frac{1}{2}L_1 \quad b = \frac{1}{4}L_1$

$\theta = 10^o \quad a + 2.5b = 24.75$ ft

$\gamma = 20^o$

Using law of cosines

$$L_2 = \sqrt{(a+b)^2 + L_1^2 - 2(a+b)L_1\cos(\theta)}$$

$L_2 = 6.425$ ft

$$\beta = \arccos\left[\frac{L_1^2 + L_2^2 - (a+b)^2}{2L_1L_2}\right]$$

$\beta = 26.484°$

$\beta_1 = \pi - (\theta + \pi - \gamma) \quad \beta_1 = 10°$

$\beta_2 = \beta - \beta_1 \quad \beta_2 = 16.484°$

SOLUTION APPROACH: FIND T THEN $A_c = T/(\sigma_U/\text{FS})$

STATICS at point H

$$\sum_H F_x = 0 \qquad T_1\sin(\beta_1) = T_2\sin(\beta_2)$$

So $\quad T_2 = T_1\dfrac{\sin(\beta_1)}{\sin(\beta_2)}$

$$\sum_H F_Y = 0 \qquad T_1\cos(\beta_1) + T_2\cos(\beta_2) = F$$

and $\quad F = W/2, \quad W = 85\text{ k}$

So $\quad T_1\left(\cos(\beta_1) + \dfrac{\sin(\beta_1)}{\sin(\beta_2)}\cos(\beta_2)\right) = F$

$$T_1 = \frac{\dfrac{W}{2}}{\left(\cos(\beta_1) + \dfrac{\sin(\beta_1)}{\sin(\beta_2)}\cos(\beta_2)\right)}$$

$T_1 = 27.042\text{ k}$

$T_2 = T_1\dfrac{\sin(\beta_1)}{\sin(\beta_2)} \qquad T_2 = 16.549\text{ k}$

COMPUTE REQUIRED CROSS-SECTIONAL AREA

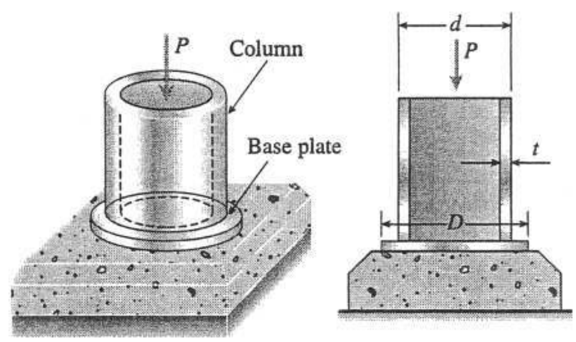
$\sigma_u = 91\text{ ksi} \qquad \text{FS} = 4 \qquad \dfrac{\sigma_u}{\text{FS}} = 22.75\text{ ksi}$

$A_c = \dfrac{T_1}{\dfrac{\sigma_u}{\text{FS}}} \qquad A_c = 1.189\text{ in.}^2 \quad \leftarrow$

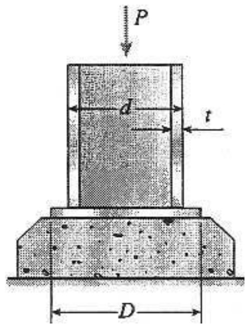
Problem 1.9-12 A steel column of hollow circular cross section is supported on a circular steel base plate and a concrete pedestal (see figure). The column has outside diameter $d = 250$ mm and supports a load $P = 750$ kN.

(a) If the allowable stress in the column is 55 MPa, what is the minimum required thickness t? Based upon your result, select a thickness for the column. (Select a thickness that is an even integer, such as 10, 12, 14, . . . , in units of millimeters.)

(b) If the allowable bearing stress on the concrete pedestal is 11.5 MPa, what is the minimum required diameter D of the base plate if it is designed for the allowable load P_{allow} that the column with the selected thickness can support?



Solution 1.9-12 Hollow circular column



$d = 250$ mm $\quad P = 750$ kN

$\sigma_{allow} = 55$ MPa (compression in column)

t = thickness of column

D = diameter of base plate

$\sigma_b = 11.5$ MPa (allowable pressure on concrete)

(a) Thickness t of the column

$$A = \frac{P}{\sigma_{allow}} \qquad A = \frac{\pi d^2}{4} - \frac{\pi}{4}(d - 2t)^2$$

$$= \frac{\pi}{4}(4t)(d - t) = \pi t(d - t)$$

$$\pi t(d - t) = \frac{P}{\sigma_{allow}}$$

$$\pi t^2 - \pi t d + \frac{P}{\sigma_{allow}} = 0$$

$$t^2 - td + \frac{P}{\pi\sigma_{allow}} = 0 \qquad \text{(Eq. 1)}$$

Substitute numerical values in Eq. (1):

$$t^2 - 250\,t + \frac{(750 \times 10^3\text{ N})}{\pi(55\text{ N/mm}^2)} = 0$$

(Note: In this eq., t has units of mm.)

$t^2 - 250t + 4{,}340.6 = 0$

Solve the quadratic eq. for t:

$t = 18.77$ mm $\qquad t_{min} = 18.8$ mm $\quad \leftarrow$

Use $t = 20$ mm $\quad \leftarrow$

(b) Diameter D of the base plate

For the column, $\qquad P_{allow} = \sigma_{allow} A$

where A is the area of the column with $t = 20$ mm.

$A = \pi t(d - t)\ P_{allow} = \sigma_{allow}\ \pi t(d - t)$

$$\text{Area of base plate} = \frac{\pi D^2}{4} = \frac{P_{allow}}{\sigma_b}$$

$$\frac{\pi D^2}{4} = \frac{\sigma_{allow}\pi t(d - t)}{\sigma_b}$$

$$D^2 = \frac{4\sigma_{allow}t(d - t)}{\sigma_b}$$

$$= \frac{4(55\text{ MPa})(20\text{ mm})(230\text{ mm})}{11.5\text{ MPa}}$$

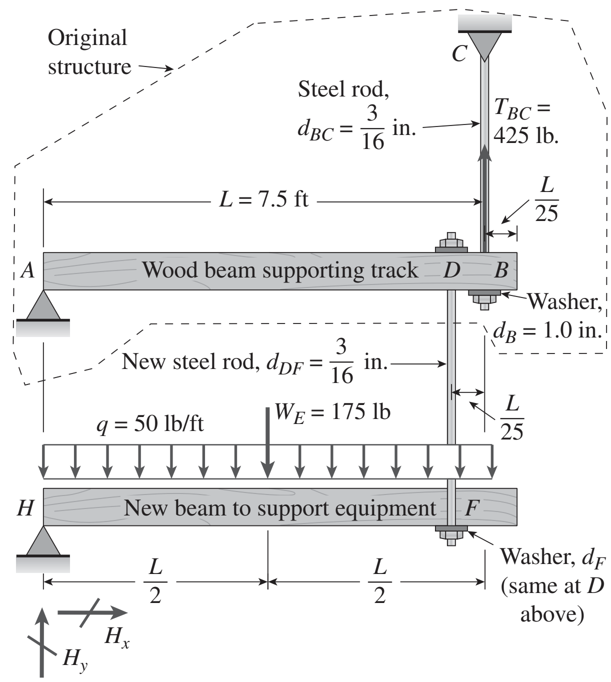
$D^2 = 88{,}000\text{ mm}^2 \qquad D = 296.6$ mm

$D_{min} = 297$ mm $\quad \leftarrow$

Problem 1.9-13 An elevated jogging track is supported at intervals by a wood beam AB (L = 7.5 ft) which is pinned at A and supported by steel rod BC and a steel washer at B. Both the rod (d_{BC} = 3/16 in.) and the washer (d_B = 1.0 in.) were designed using a rod tension force of T_{BC} = 425 lb. The rod was sized using a factor of safety of 3 against reaching the ultimate stress σ_u = 60 ksi. An allowable bearing stress σ_{ba} = 565 psi was used to size the washer at B.

Now, a small platform HF is to be suspended below a section of the elevated track to support some mechanical and electrical equipment. The equipment load is uniform load q = 50 lb/ft and concentrated load W_E = 175 lb at mid-span of beam HF. The plan is to drill a hole through beam AB at D and install the same rod (d_{BC}) and washer (d_B) at both D and F to support beam HF.

(a) Use σ_u and σ_{ba} to check the proposed design for rod DF and washer d_F; are they acceptable?
(b) Also re-check the normal tensile stress in rod BC and bearing stress at B; if either is inadequate under the additional load from platform HF, redesign them to meet the original design criteria.



Solution 1.9-13

NUMERICAL DATA

$L = 7.5(12)$ $\quad L = 90$ in. $\quad T_{BC} = 425$ lb

$\sigma_u = 60$ ksi $\quad \text{FS}_u = 3 \quad \sigma_{ba} = 0.565$ ksi

$q = \frac{50}{12} \quad q = 4.167$ lb/in. $\quad W_E = 175$ lb

$d_{BC} = \frac{3}{16}$ in. $\quad d_B = 1.0$ in.

(a) FIND FORCE IN ROD DF AND FORCE ON WASHER AT F

$$\Sigma M_H = 0 \qquad T_{DF} = \frac{W_E\frac{L}{2} + qL\frac{L}{2}}{\left(L - \frac{L}{25}\right)}$$

$$T_{DF} = 286.458 \text{ lb}$$

NORMAL STRESS IN ROD DF:

$$\sigma_{DF} = \frac{T_{DF}}{\frac{\pi}{4}d_{BC}^2}$$

$\sigma_{DF} = 10.38$ ksi $\quad$ OK—less than σ_a; rod is acceptable $\leftarrow$

$$\sigma_a = \frac{\sigma_u}{\text{FS}_u} \qquad \sigma_a = 20 \text{ ksi}$$

BEARING STRESS ON WASHER AT F:

$$\sigma_{bF} = \frac{T_{DF}}{\frac{\pi}{4}(d_B^2 - d_{BC}^2)}$$

$\sigma_{bF} = 378$ psi $\quad$ OK—less than σ_{ba}; washer is acceptable $\leftarrow$

(b) FIND NEW FORCE IN ROD BC—SUM MOMENT ABOUT A FOR UPPER FBD—THEN CHECK NORMAL STRESS IN BC and BEARING STRESS AT B

$$\Sigma M_A = 0$$

$$T_{BC2} = \frac{T_{BC}L + T_{DF}\left(L - \frac{L}{25}\right)}{L}$$

$$T_{BC2} = 700 \text{ lb}$$

REVISED NORMAL STRESS IN ROD BC:

$$\sigma_{BC2} = \frac{T_{BC2}}{\left(\frac{\pi}{4} d_{BC}{}^2\right)}$$

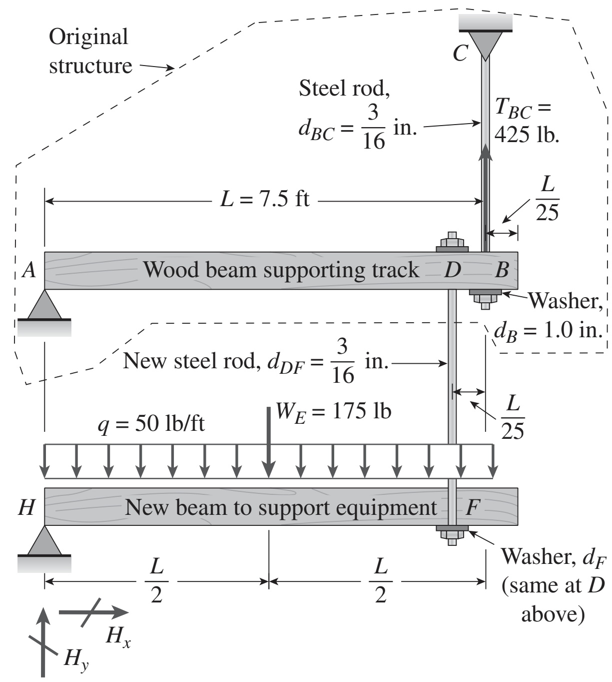
$\sigma_{BC2} = 25.352$ ksi exceeds $\sigma_a = 20$ ksi

SO RE-DESIGN ROD BC:

$$d_{BC\text{reqd}} = \sqrt{\frac{T_{BC2}}{\frac{\pi}{4}\sigma_a}}$$

$d_{BC\text{reqd}} = 0.211$ in. $d_{BC\text{reqd}} \times 16 = 3.38$

^say $4/16 = 1/4$ in. $d_{BC2} = \frac{1}{4}$ in.



RE-CHECK BEARING STRESS IN WASHER AT B:

$$\sigma_{bB2} = \frac{T_{BC2}}{\left[\frac{\pi}{4}\left(d_B^2 - d_{BC}{}^2\right)\right]}$$

$\sigma_{bB2} = 924$ psi
^ exceeds
$\sigma_{ba} = 565$ psi

SO RE-DESIGN WASHER AT B:

$$d_{B\text{reqd}} = \sqrt{\frac{T_{BC2}}{\frac{\pi}{4}\sigma_{ba}} + d_{BC}{}^2}$$

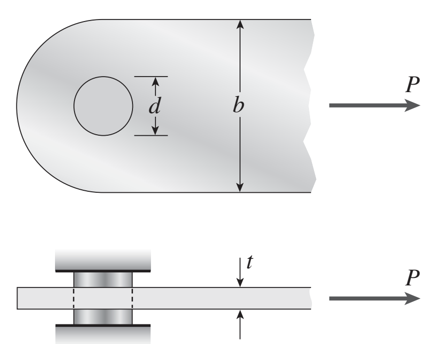
$d_{B\text{reqd}} = 1.281$ in.

use $1 - 5/16$ in washer at B: $1 + 5/16 = 1.312$ in. $\leftarrow$

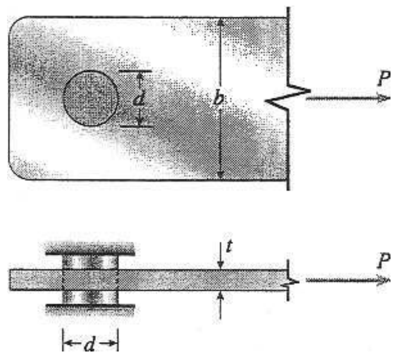
Problem 1.9-14 A flat bar of width $b = 60$ mm and thickness $t = 10$ mm is loaded in tension by a force P (see figure). The bar is attached to a support by a pin of diameter d that passes through a hole of the same size in the bar. The allowable tensile stress on the net cross section of the bar is $\sigma_T = 140$ MPa, the allowable shear stress in the pin is $\tau_S = 80$ MPa, and the allowable bearing stress between the pin and the bar is $\sigma_B = 200$ MPa.

(a) Determine the pin diameter d_m for which the load P will be a maximum.

(b) Determine the corresponding value $P_{\max}$ of the load.



Solution 1.9-14 Bar with a pin connection



$b = 60$ mm

$t = 10$ mm

d = diameter of hole and pin

$\sigma_T = 140$ MPa

$\tau_S = 80$ MPa

$\sigma_B = 200$ MPa

UNITS USED IN THE FOLLOWING CALCULATIONS:

P is in kN

σ and τ are in N/mm^2 (same as MPa)

b, t, and d are in mm

TENSION IN THE BAR

$$P_T = \sigma_T\,(\text{Net area}) = \sigma_t(t)(b - d)$$

$$= (140 \text{ MPa})(10 \text{ mm})\,(60 \text{ mm} - d)\left(\frac{1}{1000}\right)$$

$$= 1.40\,(60 - d) \qquad \text{(Eq. 1)}$$

SHEAR IN THE PIN

$$P_S = 2\tau_S A_{\text{pin}} = 2\tau_S\left(\frac{\pi d^2}{4}\right)$$

$$= 2(80 \text{ MPa})\left(\frac{\pi}{4}\right)(d^2)\left(\frac{1}{1000}\right)$$

$$= 0.040\,\pi d^2 = 0.12566 d^2 \qquad \text{(Eq. 2)}$$

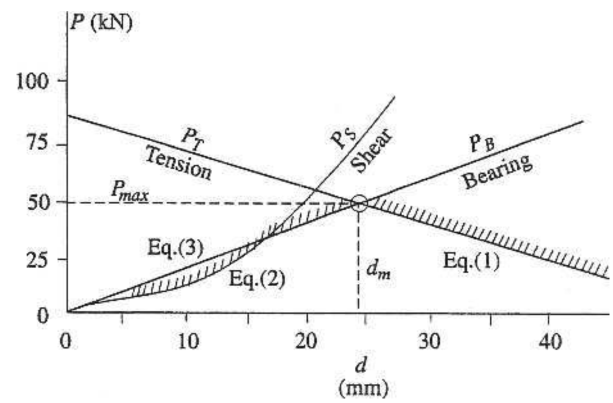
BEARING BETWEEN PIN AND BAR

$$P_B = \sigma_B\, td$$

$$= (200 \text{ MPa})(10 \text{ mm})(d)\left(\frac{1}{1000}\right)$$

$$= 2.0\, d \qquad \text{(Eq. 3)}$$

GRAPH OF EQS. (1), (2), AND (3)



(a) PIN DIAMETER d_m

$P_T = P_B$ or $1.40(60 - d) = 2.0\,d$

Solving, $d_m = \dfrac{84.0}{3.4}$ mm $= 24.7$ mm ←

(b) LOAD P_{max}

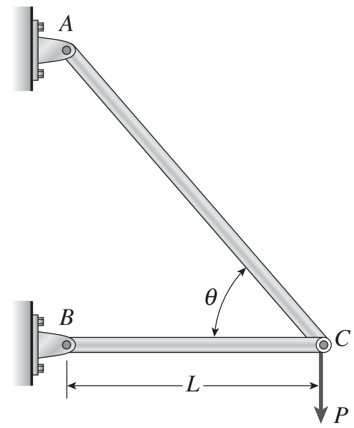
Substitute d_m into Eq. (1) or Eq. (3):

$P_{\text{max}} = 49.4$ kN ←

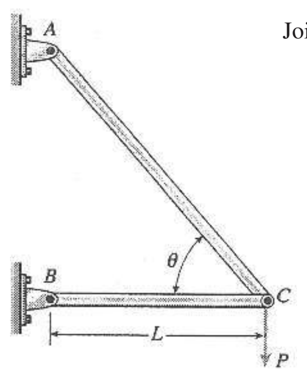
Problem 1.9-15 Two bars AC and BC of the same material support a vertical load P (see figure). The length L of the horizontal bar is fixed, but the angle θ can be varied by moving support A vertically and changing the length of bar AC to correspond with the new position of support A. The allowable stresses in the bars are the same in tension and compression.

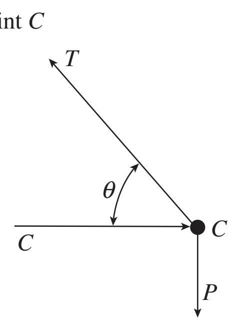
We observe that when the angle θ is reduced, bar AC becomes shorter but the cross-sectional areas of both bars increase (because the axial forces are larger). The opposite effects occur if the angle θ is increased. Thus, we see that the weight of the structure (which is proportional to the volume) depends upon the angle θ.

Determine the angle θ so that the structure has minimum weight without exceeding the allowable stresses in the bars. (*Note*: The weights of the bars are very small compared to the force P and may be disregarded.)



Solution 1.9-15 Two bars supporting a load *P*





T = tensile force in bar AC

C = compressive force in bar BC

$$\sum F_{\text{vert}} = 0 \qquad T = \frac{P}{\sin\theta}$$

$$\sum F_{\text{horiz}} = 0 \qquad C = \frac{P}{\tan\theta}$$

AREAS OF BARS

$$A_{AC} = \frac{T}{\sigma_{\text{allow}}} = \frac{P}{\sigma_{\text{allow}}\sin\theta}$$

$$A_{BC} = \frac{C}{\sigma_{\text{allow}}} = \frac{P}{\sigma_{\text{allow}}\tan\theta}$$

LENGTHS OF BARS

$$L_{AC} = \frac{L}{\cos\theta} \qquad L_{BC} = L$$

WEIGHT OF TRUSS

γ = weight density of material

$$W = \gamma(A_{AC}L_{AC} + A_{BC}L_{BC})$$

$$= \frac{\gamma PL}{\sigma_{\text{allow}}}\left(\frac{1}{\sin\theta\cos\theta} + \frac{1}{\tan\theta}\right)$$

$$= \frac{\gamma PL}{\sigma_{\text{allow}}}\left(\frac{1 + \cos^2\theta}{\sin\theta\cos\theta}\right) \qquad \text{Eq. (1)}$$

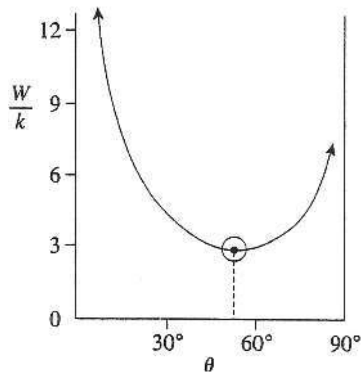
γ, P, L, and σ_{allow} are constants

W varies only with θ

Let $k = \dfrac{\gamma PL}{\sigma_{\text{allow}}}$ (k has unis of force)

$$\frac{W}{k} = \frac{1 + \cos^2\theta}{\sin\theta\cos\theta} \text{ (Nondimensional)} \qquad \text{Eq. (2)}$$

GRAPH OF EQ. (2):



ANGLE θ THAT MAKES W_A MINIMUM

Use Eq. (2)

Let $f = \dfrac{1 + \cos^2\theta}{\sin\theta\cos\theta}$

$$\frac{df}{d\theta} = 0$$

$$\frac{df}{d\theta} = \frac{(\sin\theta\cos\theta)(2)(\cos\theta)(-\sin\theta) - (1 + \cos^2\theta)(-\sin^2\theta + \cos^2\theta)}{\sin^2\theta\cos^2\theta}$$

$$= \frac{-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta}$$

SET THE NUMERATOR = 0 AND SOLVE FOR θ:

$$-\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta - \cos^4\theta = 0$$

Replace $\sin^2\theta$ by $1 - \cos^2\theta$:

$$-(1 - \cos^2\theta)(\cos^2\theta) + 1 - \cos^2\theta - \cos^2\theta - \cos^4\theta = 0$$

Combine terms to simplify the equation:

$$1 - 3\cos^2\theta = 0 \qquad \cos\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 54.7° \quad \leftarrow$$